## Package 'DTRlearn2'

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Title Statistical Learning Methods for Optimizing Dynamic Treatment Regimes

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#### Description

We provide a comprehensive software to estimate general K-stage DTRs from SMARTs with Qlearning and a variety of outcome-weighted learning methods. Penalizations are allowed for variable selection and model regularization. With the outcome-weighted learning scheme, different loss functions - SVM hinge loss, SVM ramp loss, binomial deviance loss, and L2 loss are adopted to solve the weighted classification problem at each stage; augmentation in the outcomes is allowed to improve efficiency. The estimated DTR can be easily applied to a new sample for individualized treatment recommendations or DTR evaluation.

**Depends** kernlab, MASS, Matrix, foreach, glmnet, R (>= 2.10)

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#### Description

We provide a two-stage sequential multiple assignment randomized trial (SMART) data of 150 children with ADHD mimicking a real world study. At the first stage, children were randomized to treatment of low-intensity behavioral modification (BMOD) or low-intensity methamphetamine (MED) with equal probability. At second stage, children were randomized to treatment of low-intensity MED, or high-intensity BMOD with equal probability. The primary outcome of study was children's school performance score ranging from 1 to 5 assessed at the end of the study for all participants.

#### Usage

data("adhd")

#### Format

A data frame with 150 observations on the following 11 variables.

- id IDs of the 150 children
- o11 baseline covariate coded as 0/1: diagnosed with ODD (oppositional defiant disorder) before the first-stage intervention
- o12 baseline covariate: ADHD score at the end of the previous school year (ranging from 0 to 3, larger values for fewer ADHD symptoms)
- o13 baseline covariate coded as 0/1: receiving medication during the previous school year
- o14 baseline covariate coded as 0/1: race white (coded 1) versus nonwhite (coded 0)
- a1 first-stage intervention coded as -1/1: -1 for low-intensity methamphetamine (MEDS), 1 for low-intensity behavioral modification (BMOD)
- r first-stage response indicator coded as 0/1
- o21 intermediate outcome: number of months until non-response (maximum: 8 months, NA for responders)
- o22 intermediate outcome coded as 0/1: adherence to the first-stage intervention, 1 for high adherence
- a2 second-stage intervention coded as -1/1: -1 for low-intensity BMOD + MEDS, 1 for highintensity BMOD
- y primary outcome (continuous): school performance at the end of the school year (ranging from 1 to 5, higher values reflect better performance)

#### References

Pelham Jr, W. E., & Fabiano, G. A. (2008). Evidence-based psychosocial treatments for attentiondeficit/hyperactivity disorder. Journal of Clinical Child & Adolescent Psychology, 37(1), 184-214.

#### adhd

#### owl

## Examples

owl

Integrated Outcome-weighted Learning for Estimating Optimal DTRs

#### Description

This function implements a variety of outcome-weighted learning methods for estimating general K-stage DTRs. Different loss functions - SVM hinge loss, SVM ramp loss, binomial deviance loss, and L2 loss - can be adopted to solve the weighted classification problem at each stage. Augmentation in the outcomes is allowed to improve efficiency especially when there are multiple stages with a small sample size. Cross validation is conducted to choose the best tuning parameters if any.

#### Usage

```
owl(H, AA, RR, n, K, pi='estimated', res.lasso=TRUE, loss='hinge', kernel='linear',
augment=FALSE, c=2^(-2:2), sigma=c(0.03,0.05,0.07), s=2.^(-2:2), m=4)
```

| Н  | subject history information before treatment for the K stages. It can be a vector or a matrix when only baseline information is used in estimating the DTR; otherwise, it would be a list of length K. Please standardize all the variables in H to have mean 0 and standard deviation 1 before using H as the input. See details for how to construct H. |
|----|---|
| AA | observed treatment assignments for all subjects at the K stages. It is a vector if K=1, or a list of K vectors corresponding to the K stages.   |
| RR | observed reward outcomes for all subjects at the K stages. It is a vector if K=1, or a list of K vectors corresponding to the K stages.   |
| n  | sample size, number of subjects in the dataset  |

| К         | number of stages  |
|-----------|---|
| pi        | treatment assignment probabilities of the observed treatments for all subjects at the K stages. It is a vector if K=1, or a list of K vectors corresponding to the K stages. It can be a user specified input if the treatment assignment probabilities are known. The default is pi="estimated", that is we estimate the treatment assignment probabilities based on lasso-penalized logistic regressions with $H_k$ being the predictors at each stage k.   |
| res.lasso | whether or not to use lasso penalty in the regression to take residuals for con-<br>structing the weights. The default is res.lasso=TRUE.   |
| loss      | loss function for sovling the weighted classification problem at each stage. The options are "hinge", "ramp", "logit", "logit.lasso", "l2", "l2.lasso". "hinge" and "ramp" are for the SVM hinge loss and SVM ramp loss. "logit" and "logit.lasso" are for the binomial deviance loss used in the logistic regression, where lasso penalty is applied under "logit.lasso". "l2" and "l2.lasso" are for the L2 or square loss, where lasso penalty is applied under "l2.lasso". The default is loss="hinge". |
| kernel    | kernel function to use under SVM hinge loss or SVM ramp loss. "linear" and "rbf" kernel are implemented under SVM hinge loss; "linear" kernel is implemented under SVM ramp loss. The default is kernel="linear".   |
| augment   | whether or not to use augmented outcomes at each stage. Augmentation is rec-<br>ommended when there are multiple stages and the sample size is small. The<br>default is augment=FALSE.  |
| с         | a vector specifies the values of the regularization parameter C for tuning under SVM hinge loss or SVM ramp loss. The default is $c=2^{(-2:2)}$ . In practice, a wider range of c can be specified based on the data.   |
| sigma     | a vector specifies the values of the positive parameter $\sigma$ in the RBF kernel for tuning under SVM hinge loss, i.e., when loss="hinge" and kernel="rbf". The default is sigma=c(0.03,0.05,0.07). In practice, a wider range of sigma can be specified based on the data.   |
| S         | a vector specifies the values of the slope parameter in the SVM ramp loss for tun-<br>ing, i.e., when loss="ramp" and kernel="linear". The default is c=2^(-2:2).<br>In practice, a wider range of s can be specified based on the data.  |
| m         | number of folds in the m-fold cross validation for choosing the tuning parameters c, sigma or s. It is also used for choosing the tuning parameter of the lasso penalty when res.lasso=T, loss="logit.lasso" or loss="l2.lasso" is specified. The default is m=4.   |

## Details

A patient's history information prior to the treatment at stage k can be constructed recursively as  $H_k = (H_{k-1}, A_{k-1}, R_{k-1}, X_k)$  with  $H_1 = X_1$ , where  $X_k$  is subject-specific variables collected at stage k just prior to the treatment,  $A_k$  is the treatment at stage k, and  $R_k$  is the outcome observed post the treatment at stage k. Higher order or interaction terms can also be easily incorporated in  $H_k$ , e.g.,  $H_k = (H_{k-1}, A_{k-1}, R_{k-1}, X_k, H_{k-1}A_{k-1}, R_{k-1}A_{k-1}, X_kA_{k-1})$ .

owl

## Value

A list of results is returned as an object. It contains the following attributes:

| stage1   | a list of stage 1 results,  |  |  |  |  |  |  |
|--|---|--|--|--|--|--|--|
| stageK   | a list of stage K results   |  |  |  |  |  |  |
| valuefun   | overall empirical value function under the estimated DTR  |  |  |  |  |  |  |
| benefit  | overall empirical benefit function under the estimated DTR  |  |  |  |  |  |  |
| pi   | treatment assignment probabilities of the observed treatments for each subject<br>at the K stages. It is a list of K vectors. If pi='estimated' is specified as input,<br>the estimated treatment assignment probabilities from lasso-penalized logistic<br>regressions will be returned. |  |  |  |  |  |  |
| type   | object type corresponding to the specified loss and kernel  |  |  |  |  |  |  |
| In each stage's result, a list is returned which consists of |   |  |  |  |  |  |  |
| beta0  | estimated coefficient of the intercept in the decision function   |  |  |  |  |  |  |
| beta   | estimated coefficients of $H_k$ in the decision function. It's not returned with RBF kernel under SVM hinge loss.   |  |  |  |  |  |  |
| fit  | fitted decision function for each subject   |  |  |  |  |  |  |
| probability  | estimated probability that treatment 1 (vs1) is the optimal treatment for each subject in the sample. It's calculated by $\exp(fit)/(1 + \exp(fit))$ .  |  |  |  |  |  |  |
| treatment  | the estimated optimal treatment for each subject  |  |  |  |  |  |  |
| С  | the best regularization parameter C in SVM hinge loss or SVM ramp loss, cho-<br>sen from the values specified in c via cross validation   |  |  |  |  |  |  |
| sigma  | the best parameter $\sigma$ in the RBF kernel, chosen from the values specified in sigma via cross validation   |  |  |  |  |  |  |
| S  | the best slope parameter s in the ramp loss, chosen from the values specified in s via cross validation.  |  |  |  |  |  |  |
| iter   | number of iterations under SVM ramp loss  |  |  |  |  |  |  |
| alpha1   | the solution to the Lagrangian dual problem under SVM hinge loss or SVM ramp loss. It is used for constructing the decision function on the new sample.   |  |  |  |  |  |  |
| Н  | the input H, returned only under SVM hinge loss with RBF kernel. It is used for constructing the RBF kernel on the new sample.  |  |  |  |  |  |  |

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#### References

Liu, Y., Wang, Y., Kosorok, M., Zhao, Y., & Zeng, D. (2014). Robust hybrid learning for estimating personalized dynamic treatment regimens. arXiv preprint. arXiv, 1611.

Liu, Y., Wang, Y., Kosorok, M., Zhao, Y., & Zeng, D. (2018). Augmented Outcome-weighted Learning for Estimating Optimal Dynamic Treatment Regimens. Statistics in Medicine. In press.

Zhao, Y., Zeng, D., Rush, A. J., & Kosorok, M. R. (2012). Estimating individualized treatment rules using outcome weighted learning. Journal of the American Statistical Association, 107(499), 1106-1118.

Zhao, Y. Q., Zeng, D., Laber, E. B., & Kosorok, M. R. (2015). New statistical learning methods for estimating optimal dynamic treatment regimes. Journal of the American Statistical Association, 110(510), 583-598.

#### See Also

predict.owl, sim\_Kstage, ql

#### Examples

```
# simulate 2-stage training and test sets
n train = 100
n_{test} = 500
n_cluster = 10
pinfo = 10
pnoise = 20
train = sim_Kstage(n_train, n_cluster, pinfo, pnoise, K=2)
H1_train = scale(train$X)
H2_train = scale(cbind(H1_train, train$A[[1]], H1_train * train$A[[1]]))
pi_train = list(rep(0.5, n_train), rep(0.5, n_train))
test = sim_Kstage(n_test, n_cluster, pinfo, pnoise, train$centroids, K=2)
H1_test = scale(test$X)
H2_test = scale(cbind(H1_test, test$A[[1]], H1_test * train$A[[1]]))
pi_test = list(rep(0.5, n_test), rep(0.5, n_test))
# estimate DTR with owl on the training sample
owl_train = owl(H=list(H1_train, H2_train), AA=train$A, RR=train$R, n=n_train, K=2, pi=pi_train,
    loss='hinge', augment=TRUE, m=3)
owl_train$stage1$beta
owl_train$stage1$treatment
owl_train$valuefun
# apply the estimated DTR to the test sample
owl_test = predict(owl_train, H=list(H1_test, H2_test), AA=test$A, RR=test$R, K=2, pi=pi_test)
owl_test$treatment
owl_test$valuefun
```

## Description

This function serves two purposes from a fitted "owl" object. It can recommend treatments for a new independent sample with partial or full subject features observed up to a certain stage. If subject features, treatment assignments and outcomes are fully observed in the new sample, this function can also evaluate the estimated DTR on this new sample, returning the empirical value function and benefit function.

## Usage

## S3 method for class 'owl'
predict(object, H, AA=NULL, RR=NULL, K, pi=NULL, ...)

| object | fitted "owl" object   |
|--------|---|
| Н      | subject history information before treatment at the K stages for all subjects in the new sample. It should be constructed the same way as the H in fitting the owl object. See owl for how to construct H. Partial history information is allowed - when first j (j<=K) stages of H is specified, the first j stage treatments will be recommended.   |
| AA     | observed treatment assignments at the K stages for all subjects in the new sample. It is a vector if K=1, or a list of K vectors corresponding to the K stages. If not specified, treatments will be recommended for the new sample instead of DTR evaluation. The default is AA=NULL.  |
| RR     | observed outcomes at the K stages for all subjects in the new sample. It is a vector if K=1, or a list of K vectors corresponding to the K stages. If not specified, treatments will be recommended for the new sample instead of DTR evaluation. The default is RR=NULL.   |
| К      | number of stages of H observed in the new sample  |
| pi     | treatment assignment probabilities of the observed treatments at the K stages<br>for all subjects in the new sample. It is a vector if K=1, or a list of K vectors<br>corresponding to the K stages. It can be unspecified if one is only interested in<br>treatment recommendations for the new sample. If both AA and RR are specified<br>while pi is not specified, we will estimate the treatment assignment probabilities<br>based on lasso-penalized logistic regressions with predictors being $H_k$ at each<br>stage k. The default is pi=NULL. |
|        | further arguments passed to or from other methods.  |

#### Value

| fit         | fitted decision functions at the K stages for each subject in the new sample. It is a list of K vectors.  |
|-------------|---|
| probability | estimated probability that treatment 1 (vs1) is the optimal treatment at each stage for each subject in the new sample. It's calculated by $\exp(fit)/(1 + \exp(fit))$ . It is a list of K vectors.   |
| treatment   | recommenned optimal treatments at the K stages for each subject in the new sample. It is a list of K vectors.   |
| valuefun    | overall empirical value function under the fitted DTR evaluated on the new sam-<br>ple. It is returned only when AA and RR are fully specified for the K stages.  |
| benefit     | overall empirical benefit function under the estimated DTR evaluated on the new sample. It is returned only when AA and RR are fully specified for the K stages.  |
| pi          | treatment assignment probabilities of the assigned treatments at the K stages for<br>each subject in the new sample. If pi is not specified but H and AA are specified<br>for the K stages, the estimated treatment assignment probabilities from lasso-<br>penalized logistic regressions with $H_k$ being the predictors at each stage k will<br>be returned. |

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## See Also

owl, sim\_Kstage, ql

## Examples

```
# simulate 2-stage training and test sets
n_train = 100
n_test = 500
n_cluster = 10
pinfo = 10
pnoise = 20
train = sim_Kstage(n_train, n_cluster, pinfo, pnoise, K=2)
H1_train = scale(train$X)
H2_train = scale(cbind(H1_train, train$A[[1]], H1_train * train$A[[1]]))
pi_train = list(rep(0.5, n_train), rep(0.5, n_train))
test = sim_Kstage(n_test, n_cluster, pinfo, pnoise, train$centroids, K=2)
H1_test = scale(test$X)
H2_test = scale(cbind(H1_test, test$A[[1]], H1_test * train$A[[1]]))
pi_test = list(rep(0.5, n_test), rep(0.5, n_test))
# estimate DTR with owl on the training sample
```

owl\_train = owl(H=list(H1\_train, H2\_train), AA=train\$A, RR=train\$R, n=n\_train, K=2,

## predict.ql

```
pi=pi_train, loss='hinge', augment=TRUE, m=3)
# evaluate the DTR when full information are observed on the new sample
owl_test = predict(owl_train, H=list(H1_test, H2_test), AA=test$A, RR=test$R, K=2)
owl_test$treatment
owl_test$valuefun
owl_test$valuefun
owl_test$pi
# recommned the first-stage treatments only
owl_test2 = predict(owl_train, H=H1_test, K=1)
owl_test$treatment
```

predict.ql

Predict from a Fitted "ql" Object

#### Description

This function serves two purposes from a fitted "ql" object. It can recommend treatments for a new independent sample with partial or full subject features observed up to a certain stage. If subject features, treatment assignments and outcomes are fully observed in the new sample, this function can also evaluate the fitted DTR on this new sample, returning the empirical value function and benefit function.

#### Usage

```
## S3 method for class 'ql'
predict(object, H, AA=NULL, RR=NULL, K, pi=NULL, Qopt=FALSE, Qfit=FALSE, ...)
```

| object | fitted "ql" object  |
|--------|---|
| Н      | subject history information before treatment at the K stages for all subjects in the new sample. It should be constructed the same way as the H in fitting the ql object. See ql for how to construct H. Partial history information is allowed - when first j (j<=K) stages of H is specified, the first j stage treatments will be recommended. |
| AA     | observed treatment assignments at the K stages for all subjects in the new sample. It is a vector if K=1, or a list of K vectors corresponding to the K stages. If not specified, treatments will be recommended for the new sample instead of DTR evaluation. The default is AA=NULL.  |
| RR     | observed outcomes at the K stages for all subjects in the new sample. It is a vector if K=1, or a list of K vectors corresponding to the K stages. If not specified, treatments will be recommended for the new sample instead of DTR evaluation. The default is RR=NULL.   |
| К      | number of stages of H observed in the new sample  |

| pi   | treatment assignment probabilities of the observed treatments at the K stages<br>for all subjects in the new sample. It is a vector if K=1, or a list of K vectors<br>corresponding to the K stages. It can be unspecified if one is only interested in<br>treatment recommendations for the new sample. If both AA and RR are specified<br>while pi is not specified, we will estimate the treatment assignment probabilities<br>based on lasso-penalized logistic regressions with predictors being $H_k$ at each<br>stage k. The default is pi=NULL. |
|------|---|
| Qopt | whether to output the predicted optimal Q-function. The default is Qfun=FALSE.  |
| Qfit | whether to output the estimated Q-function under the observed treatments. The default is fitted=FALSE.  |
|      | further arguments passed to or from other methods.  |

## Value

| treatment | recommended optimal treatments at the K stages for each subject in the new sample. It is a list of K vectors. If no tailoring variables are learned in the "ql" object, treatments will be assigned randomly with equal probability.  |
|-----------|---|
| valuefun  | overall empirical value function under the fitted DTR evaluated on the new sam-<br>ple. It is returned only when AA and RR are fully specified for the K stages.  |
| benefit   | overall empirical benefit function under the estimated DTR evaluated on the new sample. It is returned only when AA and RR are fully specified for the K stages.  |
| pi        | treatment assignment probabilities of the assigned treatments at the K stages for<br>each subject in the new sample. If pi is not specified but H and AA are specified<br>for the K stages, the estimated treatment assignment probabilities from lasso-<br>penalized logistic regressions with $H_k$ being the predictors at each stage k will<br>be returned. |
| Q         | the predicted optimal Q-function if Qfun=TRUE.  |
| fitted    | the estimated Q-function under the observed treatment if fitted=TRUE.   |

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## See Also

ql, sim\_Kstage, owl

## Examples

```
# simulate 2-stage training and test sets
n_train = 100
n_test = 500
n_cluster = 10
pinfo = 10
pnoise = 20
```

```
train = sim_Kstage(n_train, n_cluster, pinfo, pnoise, K=2)
H1_train = scale(train$X)
H2_train = scale(cbind(H1_train, train$A[[1]], H1_train * train$A[[1]]))
pi_train = list(rep(0.5, n_train), rep(0.5, n_train))
test = sim_Kstage(n_test, n_cluster, pinfo, pnoise, train$centroids, K=2)
H1_test = scale(test$X)
H2_test = scale(cbind(H1_test, test$A[[1]], H1_test * train$A[[1]]))
pi_test = list(rep(0.5, n_test), rep(0.5, n_test))
# estimate DTR with ql on the training sample
ql_train = ql(H=list(H1_train, H2_train), AA=train$A, RR=train$R, K=2, pi=pi_train, m=3)
# evaluate the DTR when full information are observed on the new sample
ql_test = predict(ql_train, H=list(H1_test, H2_test), AA=test$A, RR=test$R, K=2)
ql_test$treatment
ql_test$valuefun
ql_test$pi
# recommned the first-stage treatments only
ql_test2 = predict(ql_train, H=H1_test, K=1)
ql_test2$treatment
```

ql

| Q- | learning | for. | Estim | ating | $O_l$ | otimal | DTRs |
|----|----------|------|-------|-------|-------|--------|------|
|    |          |      |       |       |       |        |      |

#### Description

This function implements Q-learning for estimating general K-stage DTRs. Lasso penalty can be applied for variable selection at each stage.

#### Usage

ql(H, AA, RR, K, pi='estimated', lasso=TRUE, m=4)

| Η  | subject history information before treatment for all subjects at the K stages. It can be a vector or a matrix when only baseline information is used in estimating the DTR; otherwise, it would be a list of length K. Please standardize all the variables in H to have mean 0 and standard deviation 1 before using H as the input. See details for how to construct H. |
|----|---|
| AA | observed treatment assignments for all subjects at the K stages. It is a vector if K=1, or a list of K vectors corresponding to the K stages.   |
| RR | observed reward outcomes for all subjects at the K stages. It is a vector if K=1, or a list of K vectors corresponding to the K stages.   |
| К  | number of stages  |

| pi    | treatment assignment probabilities of the observed treatments for all subjects at the K stages. It is a vector if K=1, or a list of K vectors corresponding to the K stages. It can be a user specified input if the treatment assignment probabilities are known. The default is $pi="estimated"$ , that is we estimate the treatment assignment probabilities based on lasso-penalized logistic regressions with $H_k$ being the predictors at each stage k. |
|-------|--|
| lasso | specifies whether to add lasso penalty at each stage when fitting the model. The default is lasso=TRUE.  |
| m     | number of folds in the m-fold cross validation. It is used when res.lasso=T is specified. The default is m=4.  |

#### Details

A patient's history information prior to the treatment at stage k can be constructed recursively as  $H_k = (H_{k-1}, A_{k-1}, R_{k-1}, X_k)$  with  $H_1 = X_1$ , where  $X_k$  is subject-specific variables collected at stage k just prior to the treatment,  $A_k$  is the treatment at stage k, and  $R_k$  is the outcome observed post the treatment at stage k. Higher order or interaction terms can also be easily incorporated in  $H_k$ , e.g.,  $H_k = (H_{k-1}, A_{k-1}, R_{k-1}, X_k, H_{k-1}A_{k-1}, R_{k-1}A_{k-1}, X_kA_{k-1})$ .

## Value

A list of results is returned as an object. It contains the following attributes:

| stage1   | a list of stage 1 results,   |  |
|--|--|--|
| stageK   | a list of stage K results  |  |
| valuefun   | overall empirical value function under the estimated DTR   |  |
| benefit  | overall empirical benefit function under the estimated DTR   |  |
| pi   | treatment assignment probabilities of the assigned treatments for each subject<br>at the K stages. If pi='estimated' is specified as input, the estimated treat-<br>ment assignment probabilities from lasso-penalized logistic regressions will be<br>returned. |  |
| In each stage's result, a list is returned which consists of |  |  |
| со   | the estimated coefficients of $(1, H, A, H * A)$ , the variables in the model at this  |  |
|  | stage  |  |
| treatment  |  |  |

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sim\_Kstage

#### References

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Qian, M., & Murphy, S. A. (2011). Performance guarantees for individualized treatment rules. Annals of statistics, 39(2), 1180.

#### See Also

predict.ql, sim\_Kstage, owl

#### Examples

```
# simulate 2-stage training and test sets
n_{train} = 100
n_{test} = 500
n_cluster = 10
pinfo = 10
pnoise = 20
train = sim_Kstage(n_train, n_cluster, pinfo, pnoise, K=2)
H1_train = scale(train$X)
H2_train = scale(cbind(H1_train, train$A[[1]], H1_train * train$A[[1]]))
pi_train = list(rep(0.5, n_train), rep(0.5, n_train))
test = sim_Kstage(n_test, n_cluster, pinfo, pnoise, train$centroids, K=2)
H1_test = scale(test$X)
H2_test = scale(cbind(H1_test, test$A[[1]], H1_test * train$A[[1]]))
pi_test = list(rep(0.5, n_test), rep(0.5, n_test))
ql_train = ql(H=list(H1_train, H2_train), AA=train$A, RR=train$R, K=2, pi=pi_train, m=3)
ql_test = predict(ql_train, H=list(H1_test, H2_test), AA=test$A, RR=test$R, K=2, pi=pi_test)
```

sim\_Kstage Simulate a K-stage Sequential Multiple Assignment Randomized Trial (SMART) data

#### Description

This function simulates a K-stage SMART data with (pinfo + pnoise) baseline variables from a multivariate Gaussian distribution. The pinfo variables have variance 1 and pairwise correlation 0.2; the pnoise variables have mean 0 and are uncorrelated with each other and with the pinfo variables.

Subjects are from n\_cluster latent groups with equal sizes, and these n\_cluster groups are characterized by their differentiable means in the pinfo feature variables. Each latent group has its own optimal treatment sequence, where the optimal treatment for subjects in group g at stage k is generated as  $A^* = 2([g/(2k-1)] \mod 2) - 1$ . The assigned treatment group (1 or -1) for each subject at each stage is randomly generated with equal probability. The primary outcome is observed only at the end of the trial, which is generated as  $R = \sum_{k=1}^{K} A_k A_k^* + N(0, 1)$ .

## Usage

sim\_Kstage (n, n\_cluster, pinfo, pnoise, centroids=NULL, K)

## Arguments

| n         | sample size, should be a multiple of n_cluster.  |
|-----------|--|
| n_cluster | number of latent groups  |
| pinfo     | number of informative baseline variables   |
| pnoise    | number of non-informative baseline variables   |
| centroids | centroids of the pinfo variables for the n_cluster groups. It is a matrix of di-<br>mension n_cluster by pinfo. It's used as the means of the multivariate Gaus-<br>sians to generate the pinfo variables for the n_cluster groups. For a training<br>set, do not assign centroids, the centroids are generated randomly from $N(0,5)$<br>by the function. For a test set, one should assign the same set of centroids as the<br>training set. |
| К         | number of stages.  |

## Value

| Х         | baseline variables. It is a matrix of dimension n by (pinfo + pnoise).   |
|-----------|--|
| A         | treatment assignments for the K-stages. It is a list of K vectors.   |
| R         | outcomes of the K-stages. It is a list of K vectors. In this simulation setting, no intermediate outcomes are observed, so the first K-1 vectors are vectors of 0. |
| optA      | optimal treatments for the K-stages. It is a list of K vectors.  |
| centroids | centroids of the pinfo variables for the n_cluster groups. It is a matrix of dimension n_cluster by pinfo.   |

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## See Also

owl,ql

## Examples

```
n_train = 100
n_test = 500
n_cluster = 10
pinfo = 10
pnoise = 20
# simulate a 2-stage training set
train = sim_Kstage(n_train, n_cluster, pinfo, pnoise, K=2)
```

## sim\_Kstage

# simulate an independent 2-stage test set with the same centroids of the training set test = sim\_Kstage(n\_test, n\_cluster, pinfo, pnoise, train\$centroids, K=2)

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