

# Package ‘csmGmm’

December 3, 2024

**Type** Package

**Title** Conditionally Symmetric Multidimensional Gaussian Mixture Model

**Version** 0.3.0

**Description** Implements the conditionally symmetric multidimensional Gaussian mixture model (csmGmm) for large-scale testing of composite null hypotheses in genetic association applications such as mediation analysis, pleiotropy analysis, and replication analysis. In such analyses, we typically have J sets of K test statistics where K is a small number (e.g. 2 or 3) and J is large (e.g. 1 million). For each one of the J sets, we want to know if we can reject all K individual nulls. Please see the vignette for a quickstart guide. The paper describing these methods is ``Testing a Large Number of Composite Null Hypotheses Using Conditionally Symmetric Multidimensional Gaussian Mixtures in Genome-Wide Studies'' by Sun R, McCaw Z, & Lin X (2024, [doi:10.1080/01621459.2024.2422124](https://doi.org/10.1080/01621459.2024.2422124)). The paper is accepted and published online (but not yet in print) in the Journal of the American Statistical Association as of Dec 1 2024.

**License** GPL-3

**Encoding** UTF-8

**RoxigenNote** 7.3.1

**Imports** dplyr, mvtnorm, rlang, magrittr

**Suggests** knitr, rmarkdown

**VignetteBuilder** knitr

**NeedsCompilation** no

**Author** Ryan Sun [aut, cre]

**Maintainer** Ryan Sun <[ryansun.work@gmail.com](mailto:ryansun.work@gmail.com)>

**Repository** CRAN

**Date/Publication** 2024-12-03 19:30:02 UTC

## Contents

calc_dens_cor . . . . .	2
calc_dens_ind_2d . . . . .	3

calc_dens_ind_3d . . . . .	3
calc_dens_ind_multiple . . . . .	4
check_incongruous . . . . .	5
find_2d . . . . .	5
find_3d . . . . .	6
find_max_means . . . . .	7
symm_fit_cor_EM . . . . .	7
symm_fit_cor_EM_fulllik . . . . .	8
symm_fit_cor_EM_noAssumption . . . . .	10
symm_fit_cor_EM_rho . . . . .	11
symm_fit_ind_EM . . . . .	12
symm_fit_ind_EM_noAssumption . . . . .	13

<b>Index</b>	<b>15</b>
--------------	-----------

---

<b>calc_dens_cor</b>	<i>calc_dens_cor.R</i>
----------------------	------------------------

---

## Description

For J\*2 matrix of J sets, calculate the density of bivariate normal under fitted c-csmGmm.

## Usage

```
calc_dens_cor(x, Zmat, corMat, log = FALSE)
```

## Arguments

x	2*1 vector of means.
Zmat	J*2 matrix of test statistics.
corMat	2*2 matrix describing correlation structure of test statistics.
log	return logarithm of density

## Value

A J\*1 vector of densities for each row of Zmat.

## Examples

```
x <- c(0, 0)
Zmat <- cbind(rnorm(10^5), rnorm(10^5))
calc_dens_cor(x, Zmat, corMat = cor(Zmat))
```

---

calc\_dens\_ind\_2d      *calc\_dens\_ind.R*

---

**Description**

Calculate J bivariate normal densities (both dimensions are independent) under fitted csmGmm.

**Usage**

```
calc_dens_ind_2d(x, Zmat)
```

**Arguments**

x	2*1 vector of means.
Zmat	J*2 matrix of test statistics.

**Value**

A J\*1 vector of densities for each row of Zmat.

**Examples**

```
x <- c(0, 0)
Zmat <- cbind(rnorm(10^5), rnorm(10^5))
calc_dens_ind_2d(x, Zmat)
```

---

calc\_dens\_ind\_3d      *Calculate J trivariate normal densities (all dimensions are independent) under fitted csmGmm.*

---

**Description**

Calculate J trivariate normal densities (all dimensions are independent) under fitted csmGmm.

**Usage**

```
calc_dens_ind_3d(x, Zmat)
```

**Arguments**

x	3*1 vector of means.
Zmat	J*3 matrix of test statistics.

**Value**

A J\*1 vector of densities for each row of Zmat.

## Examples

```
x <- c(0, 0)
Zmat <- cbind(rnorm(10^5), rnorm(10^5), rnorm(10^5))
calc_dens_ind_3d(x, Zmat)
```

### calc\_dens\_ind\_multiple

*Calculate the density of K-dimensional multivariate normal (all dimensions are independent) under fitted acsGmm.*

## Description

Calculate the density of K-dimensional multivariate normal (all dimensions are independent) under fitted acsGmm.

## Usage

```
calc_dens_ind_multiple(x, Zmat)
```

## Arguments

x	K*1 vector of means.
Zmat	J*K matrix of test statistics.

## Value

A J\*1 vector of densities for each row of Zmat.

## Examples

```
x <- c(0, 0)
Zmat <- cbind(rnorm(10^5), rnorm(10^5), rnorm(10^5), rnorm(10^5))
calc_dens_ind_multiple(x, Zmat)
```

`check_incongruous`      *check\_incongruous.R*

### Description

Check the number of sets of test statistics that have a higher (less significant) lfdr value than other sets with test statistics of uniformly smaller magnitudes.

### Usage

```
check_incongruous(zMatrix, lfdrVec)
```

### Arguments

<code>zMatrix</code>	J*K vector of all test statistics.
<code>lfdrVec</code>	J*1 vector of lfdr values corresponding to each set of test statistics.

### Value

A vector with all the indices of all sets that have a higher lfdr value those a set with smaller test statistic magnitudes.

### Examples

```
zMatrix <- cbind(rnorm(10^4), rnorm(10^4))
lfdrVec <- runif(10^4)
check_incongruous(zMatrix = zMatrix, lfdrVec = lfdrVec)
```

`find_2d`

*Tells if row x if allTestStats is an incongruous result (has a higher lfdr than a set of test statistics with lower magnitudes). For K=2 case.*

### Description

Tells if row x if allTestStats is an incongruous result (has a higher lfdr than a set of test statistics with lower magnitudes). For K=2 case.

### Usage

```
find_2d(x, allTestStats)
```

### Arguments

<code>x</code>	Scalar, which row of allTestStats to check.
<code>allTestStats</code>	J*K vector of all test statistics.

**Value**

A scalar denoting the number of sets with lower lfdr and test statistics of lower magnitude. 0 means congruous result.

**Examples**

```
zMatrix <- cbind(rnorm(10^4), rnorm(10^4))
find_2d(x = 5, allTestStats = zMatrix)
```

**find\_3d**

*Tells if row x if allTestStats is an incongruous result (has a higher lfdr than a set of test statistics with lower magnitudes). For K=3 case.*

**Description**

Tells if row x if allTestStats is an incongruous result (has a higher lfdr than a set of test statistics with lower magnitudes). For K=3 case.

**Usage**

```
find_3d(x, allTestStats)
```

**Arguments**

- x               Scalar, which row of allTestStats to check.
- allTestStats    J\*K vector of all test statistics.

**Value**

A scalar denoting the number of sets with lower lfdr and test statistics of lower magnitude. 0 means congruous result.

**Examples**

```
zMatrix <- cbind(rnorm(10^4), rnorm(10^4), rnorm(10^4))
find_3d(x = 5, allTestStats = zMatrix)
```

find_max_means	<i>find_max_means.R</i>
----------------	-------------------------

### Description

Find maximum means for each dimension in null settings.

### Usage

```
find_max_means(muInfo)
```

### Arguments

muInfo	A list with $2^K$ elements, where each element is a matrix with K rows and Mb columns.
--------	--

### Value

A K\*1 vector of the maximum means for each dimension under the null.

### Examples

```
initMuList <- list(matrix(data=0, nrow=2, ncol=1), matrix(data=c(0, 3, 0, 6), nrow=2),
matrix(data=c(3, 0, 6, 0), nrow=2), matrix(data=c(8, 8), nrow=2))
find_max_means(initMuList)
```

symm_fit_cor_EM	<i>symm_fit_cor.R</i>
-----------------	-----------------------

### Description

Fit the correlated csmGmm for sets of correlated elements. Currently restricted to K=2.

### Usage

```
symm_fit_cor_EM(
  testStats,
  corMat,
  initMuList,
  initPiList,
  eps = 10^(-5),
  checkpoint = TRUE
)
```

## Arguments

testStats	J*K matrix of test statistics where J is the number of sets and K is number of elements in each set.
corMat	K*K matrix that describes the correlation structure of each set.
initMuList	List of 2^K elements where each element is a matrix with K rows and number of columns equal to the number of possible mean vectors for that binary group.
initPiList	List of 2^K elements where each element is a vector with number of elements equal to the number of possible mean vectors for that binary group.
eps	Scalar, stop the EM algorithm when L2 norm of difference in parameters is less than this value.
checkpoint	Boolean, set to TRUE to print iterations of EM.

## Value

A list with the elements:

muInfo	List with same dimensions as initMuList, holds the final mean parameters.
piInfo	List with same dimensions as initPiList, holds the final probability parameters.
iter	Number of iterations run in EM algorithm.
lfdrResults	J*1 vector of all lfdr statistics.

## Examples

```
set.seed(0)
corMat <- matrix(data=c(1, 0.3, 0.3, 1), nrow=2)
testStats <- rbind(mvtnorm::rmvnorm(n=200, mean=c(3, 0), sigma=corMat),
mvtnorm::rmvnorm(n=200, mean=c(0, 4), sigma=corMat),
mvtnorm::rmvnorm(n=100, mean=c(7, 7), sigma=corMat),
mvtnorm::rmvnorm(n=10^4 - 500, mean=c(0, 0), sigma=corMat))
initMuList <- list(matrix(data=0, nrow=2, ncol=1), matrix(data=c(0, 3), nrow=2),
matrix(data=c(4, 0), nrow=2), matrix(data=c(5, 5), nrow=2))
initPiList <- list(c(0.9), c(0.04), c(0.04), c(0.02))
results <- symm_fit_cor_EM(testStats = testStats, corMat = cor(testStats),
initMuList = initMuList, initPiList = initPiList)
```

symm\_fit\_cor\_EM\_fulllik  
symm\_fit\_cor\_fulllik.R

## Description

Full likelihood, block correlation, blocks of size 2

**Usage**

```
symm_fit_cor_EM_fulllik(
  testStats,
  corMat,
  initMuList,
  initPiList,
  eps = 10^(-5),
  checkpoint = TRUE
)
```

**Arguments**

<code>testStats</code>	$J*K$ matrix of test statistics where $J$ is the number of sets and $K$ is number of elements in each set.
<code>corMat</code>	$K*K$ matrix that describes the correlation structure of each 2 by 2 block.
<code>initMuList</code>	List of $2^K$ elements where each element is a matrix with $K$ rows and number of columns equal to the number of possible mean vectors for that binary group.
<code>initPiList</code>	List of $2^K$ elements where each element is a vector with number of elements equal to the number of possible mean vectors for that binary group.
<code>eps</code>	Scalar, stop the EM algorithm when L2 norm of difference in parameters is less than this value.
<code>checkpoint</code>	Boolean, set to TRUE to print iterations of EM.

**Value**

A list with the elements:

<code>muInfo</code>	List with same dimensions as <code>initMuList</code> , holds the final mean parameters.
<code>piInfo</code>	List with same dimensions as <code>initPiList</code> , holds the final probability parameters.
<code>iter</code>	Number of iterations run in EM algorithm.
<code>lfdrResults</code>	$J*1$ vector of all lfdr statistics.

**Examples**

```
set.seed(0)
testStats <- cbind(rnorm(10^4), rnorm(10^4))
testStats[1:100, 1] <- rnorm(100, mean=3)
testStats[101:200, 1] <- rnorm(100, mean=5)
testStats[201:300, 2] <- rnorm(100, mean=4)
testStats[301:400, 1:2] <- rnorm(200, mean=7)
initMuList <- list(matrix(data=0, nrow=2, ncol=1), matrix(data=c(0, 3), nrow=2, ncol=1),
matrix(data=c(3, 0), nrow=2, ncol=1), matrix(data=c(6, 6), nrow=2, ncol=1))
initPiList <- list(c(0.9), c(0.04), c(0.04), c(0.02))
results <- symm_fit_cor_EM_fulllik(testStats = testStats, corMat=diag(c(1,1)),
initMuList = initMuList, initPiList = initPiList)
```

**symm\_fit\_cor\_EM\_noAssumption**  
*symm\_fit\_cor\_noAssumption.R*

## Description

Fit the correlated csmGmm for sets of correlated elements, but we don't assume that the means in the composite alternative are greater in magnitude than those in the composite null.

## Usage

```
symm_fit_cor_EM_noAssumption(
  testStats,
  corMat,
  initMuList,
  initPiList,
  eps = 10^(-5),
  checkpoint = TRUE
)
```

## Arguments

<code>testStats</code>	J*K matrix of test statistics where J is the number of sets and K is number of elements in each set.
<code>corMat</code>	K*K matrix that describes the correlation structure of each set.
<code>initMuList</code>	List of 2^K elements where each element is a matrix with K rows and number of columns equal to the number of possible mean vectors for that binary group.
<code>initPiList</code>	List of 2^K elements where each element is a vector with number of elements equal to the number of possible mean vectors for that binary group.
<code>eps</code>	Scalar, stop the EM algorithm when L2 norm of difference in parameters is less than this value.
<code>checkpoint</code>	Boolean, set to TRUE to print iterations of EM.

## Value

A list with the elements:

<code>muInfo</code>	List with same dimensions as initMuList, holds the final mean parameters.
<code>piInfo</code>	List with same dimensions as initPiList, holds the final probability parameters.
<code>iter</code>	Number of iterations run in EM algorithm.
<code>lfdrResults</code>	J*1 vector of all lfdr statistics.

## Examples

```
set.seed(0)
corMat <- matrix(data=c(1, 0.3, 0.3, 1), nrow=2)
testStats <- rbind(mvtnorm::rmvnorm(n=200, mean=c(3, 0), sigma=corMat),
mvtnorm::rmvnorm(n=200, mean=c(0, 4), sigma=corMat),
mvtnorm::rmvnorm(n=100, mean=c(7, 7), sigma=corMat),
mvtnorm::rmvnorm(n=10^4 - 500, mean=c(0, 0), sigma=corMat))
initMuList <- list(matrix(data=0, nrow=2, ncol=1), matrix(data=c(0, 3), nrow=2),
matrix(data=c(4, 0), nrow=2), matrix(data=c(5, 5), nrow=2))
initPiList <- list(c(0.9), c(0.04), c(0.04), c(0.02))
results <- symm_fit_cor_EM_noAssumption(testStats = testStats,
corMat = cor(testStats), initMuList = initMuList, initPiList = initPiList)
```

symm\_fit\_cor\_EM\_rho      *symm\_fit\_cor\_rho.R*

## Description

Fit the correlated csmGmm for sets of correlated elements. Also fits the correlation parameter in EM algorithm.

## Usage

```
symm_fit_cor_EM_rho(
  testStats,
  initRho,
  initMuList,
  initPiList,
  eps = 10^(-5),
  checkpoint = TRUE
)
```

## Arguments

testStats	J*K matrix of test statistics where J is the number of sets and K is number of elements in each set.
initRho	Initial value of rho, any reasonable guess should be ok.
initMuList	List of 2^K elements where each element is a matrix with K rows and number of columns equal to the number of possible mean vectors for that binary group.
initPiList	List of 2^K elements where each element is a vector with number of elements equal to the number of possible mean vectors for that binary group.
eps	Scalar, stop the EM algorithm when L2 norm of difference in parameters is less than this value.
checkpoint	Boolean, set to TRUE to print iterations of EM.

**Value**

A list with the elements:

<code>muInfo</code>	List with same dimensions as <code>initMuList</code> , holds the final mean parameters.
<code>piInfo</code>	List with same dimensions as <code>initPiList</code> , holds the final probability parameters.
<code>iter</code>	Number of iterations run in EM algorithm.
<code>lfdrResults</code>	$J \times 1$ vector of all lfdr statistics.

**Examples**

```
set.seed(0)
corMat <- matrix(data=c(1, 0.3, 0.3, 1), nrow=2)
testStats <- rbind(mvtnorm::rmvnorm(n=200, mean=c(3, 0), sigma=corMat),
mvtnorm::rmvnorm(n=200, mean=c(0, 4), sigma=corMat),
mvtnorm::rmvnorm(n=100, mean=c(7, 7), sigma=corMat),
mvtnorm::rmvnorm(n=10^4 - 500, mean=c(0, 0), sigma=corMat))
initMuList <- list(matrix(data=0, nrow=2, ncol=1), matrix(data=c(0, 3), nrow=2),
matrix(data=c(4, 0), nrow=2), matrix(data=c(5, 5), nrow=2))
initPiList <- list(c(0.9), c(0.04), c(0.04), c(0.02))
results <- symm_fit_cor_EM_rho(testStats = testStats,
initRho = 0.1, initMuList = initMuList, initPiList = initPiList)
```

*symm\_fit\_ind\_EM*      *symm\_fit\_ind.R*

**Description**

Fit the conditionally symmetric multidimensional Gaussian mixture model for sets of independent elements

**Usage**

```
symm_fit_ind_EM(
  testStats,
  initMuList,
  initPiList,
  sameDirAlt = FALSE,
  eps = 10^{(-5)},
  checkpoint = TRUE
)
```

## Arguments

testStats	J*K matrix of test statistics where J is the number of sets and K is number of elements in each set.
initMuList	List of 2^K elements where each element is a matrix with K rows and number of columns equal to the number of possible mean vectors for that binary representation.
initPiList	List of 2^K elements where each element is a vector with number of elements equal to the number of possible mean vectors for that binary representation.
sameDirAlt	Boolean, set to TRUE for replication testing, which uses a smaller alternative space.
eps	Scalar, stop the EM algorithm when L2 norm of difference in parameters is less than this value.
checkpoint	Boolean, set to TRUE to print iterations of EM

## Value

A list with the elements:

muInfo	List with same dimensions as initMuList, holds the final mean parameters.
piInfo	List with same dimensions as initPiList, holds the final mixture proportions
iter	Number of iterations run in EM algorithm.
lfdrResults	J*1 vector of all lfdr statistics.

## Examples

```
set.seed(0)
testStats <- cbind(rnorm(10^4), rnorm(10^4))
testStats[1:200, 1] <- rnorm(100, mean=3)
testStats[201:400, 1] <- rnorm(100, mean=5)
testStats[401:600, 2] <- rnorm(100, mean=3)
testStats[601:800, 2] <- rnorm(100, mean=5)
testStats[801:1000, 1:2] <- rnorm(200, mean=7)
initMuList <- list(matrix(data=0, nrow=2, ncol=1), matrix(data=c(0, 3, 0, 5), nrow=2, ncol=2),
matrix(data=c(3, 0, 5, 0), nrow=2, ncol=2), matrix(data=c(7, 7), nrow=2, ncol=1))
initPiList <- list(c(0.9), c(0.02, 0.02), c(0.02, 0.02), c(0.02))
results <- symm_fit_ind_EM(testStats = testStats, initMuList = initMuList, initPiList = initPiList)
```

## Description

Fit the conditionally symmetric multidimensional Gaussian mixture model for sets of independent elements, but we don't assume that the means in the composite alternative are greater in magnitude than those in the composite null.

**Usage**

```
symm_fit_ind_EM_noAssumption(
  testStats,
  initMuList,
  initPiList,
  sameDirAlt = FALSE,
  eps = 10^(-5),
  checkpoint = TRUE
)
```

**Arguments**

<code>testStats</code>	$J*K$ matrix of test statistics where $J$ is the number of sets and $K$ is number of elements in each set.
<code>initMuList</code>	List of $2^K$ elements where each element is a matrix with $K$ rows and number of columns equal to the number of possible mean vectors for that binary representation.
<code>initPiList</code>	List of $2^K$ elements where each element is a vector with number of elements equal to the number of possible mean vectors for that binary representation.
<code>sameDirAlt</code>	Boolean, set to TRUE for replication testing, which uses a smaller alternative space.
<code>eps</code>	Scalar, stop the EM algorithm when L2 norm of difference in parameters is less than this value.
<code>checkpoint</code>	Boolean, set to TRUE to print iterations of EM

**Value**

A list with the elements:

<code>muInfo</code>	List with same dimensions as <code>initMuList</code> , holds the final mean parameters.
<code>piInfo</code>	List with same dimensions as <code>initPiList</code> , holds the final mixture proportions
<code>iter</code>	Number of iterations run in EM algorithm.
<code>lfdrResults</code>	$J*1$ vector of all lfdr statistics.

**Examples**

```
set.seed(0)
testStats <- cbind(rnorm(10^4), rnorm(10^4))
testStats[1:200, 1] <- rnorm(100, mean=3)
testStats[201:400, 1] <- rnorm(100, mean=5)
testStats[401:600, 2] <- rnorm(100, mean=3)
testStats[601:800, 2] <- rnorm(100, mean=5)
testStats[801:1000, 1:2] <- rnorm(200, mean=7)
initMuList <- list(matrix(data=0, nrow=2, ncol=1), matrix(data=c(0, 3, 0, 5), nrow=2, ncol=2),
matrix(data=c(3, 0, 5, 0), nrow=2, ncol=2), matrix(data=c(7, 7), nrow=2, ncol=1))
initPiList <- list(c(0.9), c(0.02, 0.02), c(0.02, 0.02), c(0.02))
results <- symm_fit_ind_EM_noAssumption(testStats = testStats,
initMuList = initMuList, initPiList = initPiList)
```

# Index

calc\_dens\_cor, 2  
calc\_dens\_ind\_2d, 3  
calc\_dens\_ind\_3d, 3  
calc\_dens\_ind\_multiple, 4  
check\_incongruous, 5  
  
find\_2d, 5  
find\_3d, 6  
find\_max\_means, 7  
  
symm\_fit\_cor\_EM, 7  
symm\_fit\_cor\_EM\_fulllik, 8  
symm\_fit\_cor\_EM\_noAssumption, 10  
symm\_fit\_cor\_EM\_rho, 11  
symm\_fit\_ind\_EM, 12  
symm\_fit\_ind\_EM\_noAssumption, 13