# Package 'parsec'

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parsec-package

#### Description

The package implements tools for the analysis of partially ordered data, with a particular focus on the evaluation of multidimensional systems of indicators and on the analysis of poverty.

Its main objective is to provide socio-economic scholars with an integrated set of elementary functions for multidimensional evaluation, based on ordinal information. In particular, it provides functions for data management and basic analysis of partial orders as well as other functions for the evaluation and application of both the poset-based approach and a more classic counting method.

#### Author(s)

A, Arcagni M, Fattore

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#### Examples

```
# a simple example of package application #
****
# definition of the variables by their number of grades
variables <- c(2, 2, 2)
# definition of the threshold
threshold <- c("112", "211")
# extraction of all of the possible profiles from variables; the
# function returns an object of class "wprof", weighted profiles: by default,
# weigths/frequencies are set equal to 1
profiles <- var2prof(varlen = variables)</pre>
# the following function creates matrices describing the poset, and
# provides all the results related to it
eval <- evaluation(profiles, threshold, nit = 10^5, maxint = 10^3)
# The results can then be summarized
summary(summary(eval))
```

# a method of the plot function returns the Hasse diagram, a frequency # distribution of the threshold, the identification function, the rank # distribution of each profile through a barplot, and the relative gap. plot(eval)

```
*****
# definition of the variables and of the corresponding profiles
v1 <- as.ordered(c("a", "b", "c", "d"))</pre>
v2 <- 1:3
prof <- var2prof(varmod = list(v1 = as.ordered(c("a", "b", "c", "d")), v2 = 1:3))</pre>
np <- nrow(prof$profiles)</pre>
# definition of different distributions over the set of profiles
k <- 10 # number of populations
set.seed(0)
populations <- as.data.frame(lapply(1:k, function(x) round(runif(np)*100)))</pre>
rownames(populations) <- rownames(prof$profiles)</pre>
names(populations) <- paste0("P", 1:k)</pre>
prof
populations
# evaluation of the fuzzy first order dominance
res <- FFOD(profiles = prof, distributions = populations)</pre>
res
# rank stablity analysis
res <- rank_stability(res)</pre>
res
# graphical representation
plot(res)
```

AF

#### **OPHI** counting approach

#### Description

The function implements the OPHI counting approach, in a single call. The implementation is limited to ordinal attributes.

#### Usage

```
AF(y, ...)
## Default S3 method:
AF(y, z, w=rep(1, ncol(y)), k=sum(w), freq=rep(1, nrow(y)), ...)
## S3 method for class 'wprof'
AF(y, ...)
```

#### Arguments

У	matrix of profiles, possibly substituted by an object of class wprof.
Z	vector of attribute cutoffs.

w	variables' weights.
k	overall cutoff.
freq	profiles' frequencies; the argument can be omitted if y is an object of class wprof
	any of the above.

# Value

An object of S3 class ophi containing all the outputs related to the OPHI counting approach. The object is a list comprising:

У	matrix of profiles,
freq	profiles' frequencies,
d	number of variables
n	number of observations (sum of frequencies),
z	vector of cutoffs,
k	overall cutoff,
rho	function comparing profiles to the vector of cutoffs,
rho_k	function comparing profiles to the overall cutoff, by weighting variables,
g0	profile-variable matrix reporting the output of function rho,
С	censored vector of deprivation counts,
Z_k	boolean vector identifying deprived profiles, according to the specified cutoffs,
q	number of poor statstical units in the population,
Н	head count ratio, i.e. $q/n$ , where n is the number of statistical units in the population,
A	average deprivation share,
MØ	adjusted headcount ratio.

# References

Alkire S., Foster J. (2011), Counting and multidimensional poverty measurement, Journal of Public Economics, 96(7-8), 476-487.

```
vl <- c(2, 3, 3, 2)
prof <- var2prof(varlen = vl)
res <- AF(prof, z = c(1, 2, 1, 1), k = 1)
res</pre>
```

AF2threshold

#### Description

The function computes the threshold in the profile poset, which makes the poset approach equivalent to the AF counting approach, described in argument mpi.

#### Usage

AF2threshold(mpi, prof, zeta = NULL)

#### Arguments

mpi	an object of class ophi, see AF for details.
prof	an object of class wprof.
zeta	an object of class incidence.

#### See Also

AF

```
vl <- c(2, 3, 2)
prof <- var2prof(varlen = vl, labtype = "progressive")
res <- AF(prof, z = c(1, 2, 1), k = 1)
thr <- AF2threshold(res, prof)
plot(prof, col = 1 + thr, lwd = 1 + res$c,
    main = "Comparison between OPHI and parsec",
    sub = "bold: deprived profiles identified by OPHI, red: parsec threshold")
eval <- evaluation(prof, thr, maxint = 10^4, nit = 10^7)
ord <- order(eval$idn_f, res$c)
plot(eval$idn_f[ord], col = "red", lwd=2, type = "1", xlab="",
    ylab = "", axes = FALSE, frame.plot = TRUE,
    main = "Comparison between OPHI and parsec",
    sub = "red: identification function, black: OPHI deprived profiles")
points(res$c[ord], type="1", lwd=2)
axis(2)</pre>
```

antisymmetry *antisymmetry* 

#### Description

The function checks whether boolean square matrix m represents an antisymmetric binary relation.

#### Usage

antisymmetry(m)

#### Arguments

m

a square matrix.

#### See Also

transitivity, binary, reflexivity, is.preorder, is.partialorder, validate.partialorder.incidence

#### Examples

```
M <- c(TRUE, FALSE, FALSE, FALSE, TRUE, TRUE, TRUE, FALSE, FALSE, TRUE, FALSE, TRUE, FALSE, TRUE, TRUE, TRUE, TRUE)
M <- matrix(M, 4, 4)
rownames(M) <- colnames(M) <- LETTERS[1:4]</pre>
```

antisymmetry(M)

average\_ranks Average Ranks

#### Description

The function evaluates the average rank, and other distribution details, for each element of the poset.

# Usage

```
average_ranks(x, ...)
## S3 method for class 'cover'
average_ranks(x, level = 0.9, error = 10^(-5), ...)
## S3 method for class 'incidence'
average_ranks(x, level = 0.9, error = 10^(-5), ...)
```

#### Arguments

х	an incidence or cover matrix representing a partial order.
level	coverage probability of the rank intervals.
error	the "distance" from uniformity in the sampling distribution of linear extensions used to evaluate the average ranks. See idn for details.
	any of above.

#### Details

The function computes the rank distribution for each element of the poset, through function idn. Next, it checks whether there are any equivalent profiles, using function equivalences, and makes their rank distribution equal. Finally it provides a dataframe comprising, for each element of the poset: the average rank avrg, the extremes inf and sup of the rank interval, the effective coverage probability of the rank interval prob, the estimated minimum and maximum rank values (min and max) and the rank range.

The output is a dataframe of class average\_ranks /for which a method of function plot is available. See plot.average\_ranks for details).

## Value

A dataframe of class average\_ranks whose columns are:

# Author(s)

Fattore M., Arcagni A.

## See Also

idn, equivalences, plot.average\_ranks

```
profiles <- var2prof(varlen = c(3, 2, 2))
Z <- getzeta(profiles)
res <- average_ranks(Z)
plot(res)</pre>
```

binary

#### Description

The function checks whether square matrix m represents a binary relation.

#### Usage

binary(m)

# Arguments m

a square matrix.

# See Also

transitivity, reflexivity, antisymmetry, is.preorder, is.partialorder, validate.partialorder.incidence

#### Examples

```
M <- c(TRUE, FALSE, FALSE, FALSE, TRUE, TRUE, TRUE, FALSE, TRUE, FALSE, TRUE, FALSE, TRUE, TRUE, TRUE, TRUE)
M <- matrix(M, 4, 4)
rownames(M) <- colnames(M) <- LETTERS[1:4]</pre>
```

binary(M)

colevels

Colevels of a poset

#### Description

The function returns colevels associated to poset elements.

#### Usage

```
colevels(y)
```

#### Arguments

у

an object of class cover or incidence.

depths

# Examples

```
vl <- c(3, 2, 4)
prof <- var2prof(varlen = vl)
Z <- getzeta(prof)
colevels(Z)</pre>
```

cover2incidence cover2incidence

# Description

The function computes the incidence matrix of a poset from its cover matrix.

# Usage

```
cover2incidence(g)
```

#### Arguments

g

a cover matrix, an object of class cover

# Value

The function returns the corresponding incidence matrix, an object of class incidence.

# See Also

incidence2cover

depths

Depths

# Description

The function computes the depths of poset elements.

# Usage

```
depths(z)
```

#### Arguments

z

an object of class cover or incidence.

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#### downset

# Examples

```
vl <- c(3, 2, 4)
prof <- var2prof(varlen = vl)
Z <- getzeta(prof)
depths(Z)</pre>
```

downset

downset

# Description

The function computes a boolean vector identifying the poset elements below (or equal to) at least one element of the input subset Q.

# Usage

```
downset(z, ...)
## S3 method for class 'cover'
downset(z, ...)
## S3 method for class 'incidence'
downset(z, Q = NULL, ...)
```

#### Arguments

Z	a cover, or an incidence, matrix of S3 class cover or incidence respectively
Q	vector of indices identifying a subset of poset profiles
	any of the above.

```
z <- getzeta(var2prof(varlen = c(2, 2, 2)))
plot(z, col = 1 + c(1, 1, 0, 0, 1, 0, 0, 0) + c(0, 0, 0, 2, 0, 0, 2, 2), lwd = 2)
Q <- c(4, 7, 8)
rownames(z)[Q]
downset(z, Q)
Q <- c("211", "112", "111")</pre>
```

drawedges

# Description

Graphical function called by plot.cover to draw the edges of the Hasse diagram representing the input cover matrix C.

# Usage

drawedges(C, vertices, ...)

# Arguments

С	cover matrix.
vertices	coordinates of the vertices obtained by function vertices.
	line parameters, see graphics{lines}.

# See Also

plot.cover, vertices, graphics{lines}

equivalences	Equivalence classes in a poset.	
--------------	---------------------------------	--

# Description

The function computes the set of poset elements sharing the same upset and downset.

# Usage

```
equivalences(x)
```

# Arguments

x an object of class incidence or cover.

# Value

The function computes a vector assigning an equivalence class to each profile. The vector is of class factor.

## Author(s)

Arcagni A.

## evaluation

#### Examples

```
Lmbd <- getlambda(A > B, A > C, B > D, A > E, B > E, C > F, C > G)
res <- equivalences(Lmbd)
equivalence_classes <- levels(res)
colrs <- sapply(res, function(x) which(equivalence_classes == x)) + 1
plot(Lmbd, col = colrs, lwd = 2)</pre>
```

evaluation

Multidimensional evaluation on posets

# Description

Given a partial order (arguments profiles and/or zeta) and a selected threshold, the function returns an object of S3 class parsec, comprising the identification function and different severity measures, computed by uniform sampling of the linear extensions of the poset, through a C implementation of the Bubley - Dyer (1999) algorithm.

#### Usage

```
evaluation(
    profiles = NULL,
    threshold,
    error = 10^{(-3)},
    zeta = getzeta(profiles),
    weights = {
        if (!is.null(profiles))
            profiles$freq
        else rep(1, nrow(zeta))
    },
    distances = {
        n <- nrow(zeta)</pre>
        matrix(1, n, n) - diag(1, n)
    },
    linext = lingen(zeta),
    nit = floor({
        n <- nrow(zeta)</pre>
        n^{5} * \log(n) + n^{4} * \log(error^{-1}))
    }),
    maxint = 2^{31} - 1,
    inequality = FALSE
)
inequality(profiles = NULL, zeta = getzeta(profiles), ...)
```

# Arguments

profiles	an object of S3 class wprof.
threshold	a vector identifying the threshold. It can be a vector of indexes (numeric), a vector of profile names (character) or a boolean vector of length equal to the number of profiles. Function inequality does not require its definition since its results do not depend on it.
error	the "distance" from uniformity in the sampling distribution of linear extensions.
zeta	the incidence matrix of the poset. An object of S3 class incidence. By default, extracted from profiles.
weights	weights assigned to profiles. If the argument profiles is not NULL, weights are by default set equal to profile frequencies, otherwise they are set equal to 1.
distances	matrix of distances between pairs of profiles. The matrix must be square, with dimensions equal to the number of profiles. Even if the poset is complete, the distance between two profiles is computed only if one profile covers the other.
linext	the linear extension initializing the sampling algorithm. By default, it is gener- ated by lingen(zeta). Alternatively, it can be provided by the user through a vector of profile positions.
nit	Number of ITerations in the Bubley-Dyer algorithm, by default evaluated from a formula of Karzanov and Khachiyan based on the number of profiles and the argument error (see Bubley and Dyer, 1999).
maxint	Maximum integer. By default the maximum integer obtainable in a 32bit system. This argument is used to group iterations and run the compiled C code more times, so as to avoid memory indexing problems. Users can set a lower value to maxint in case of low RAM availability.
inequality	boolean parameter (by default FALSE) to make the evaluation function return also a measure of inequality (which can make computations quite lengthy). It is TRUE in function inequality and can not be modified.
	further optional graphical parameters. See plot.default.

# Value

profiles	an object of S3 class wprof reporting poset profiles and their associated frequen-
	cies (number of statistical units in each profile).
number_of_profi	les
	number of profiles.
number_of_varia	bles
	number of variables.
incidence	S3 class incidence, incidence matrix of the poset.
cover	S3 class cover, cover matrix of the poset.
threshold	boolean vector specifying whether a profile belongs to the threshold.
number_of_itera	itions
	number of iterations performed by the Bubley-Dyer algorithm.
rank_dist	matrix reporting by rows the relative frequency distributions of the ranks of each profile, over the set of sampled linear extensions.

#### evaluation

thr_dist	vector reporting the relative frequency a profile is used as threshold in the sam- pled linear extensions.	
prof_w	vector of weights assigned to each profile.	
edg_w	matrix of distances between profiles, used to evaluate the gap measures.	
idn_f	vector reporting the identification function, computed as the fraction of sampled linear extensions where a profile is in the downset of the threshold.	
svr_abs	vector reporting, for each profile, the average graph distance from the first profile above all threshold elements, over the sampled linear extensions. In each linear extension, the distance is set equal to 0 for profiles above the threshold.	
svr_rel	equal to svr_abs divided by its maximum, that is svr_abs of the minimal element in the linear extension.	
wea_abs	vector reporting, for each profile, the average graph distance from the maximum threshold element, over the sampled linear extensions. In each linear extension, the distance is set equal to 0 for profiles in the downset of threshold elements.	
wea_rel	the previous absolute distance is divided by its maximum possible value, that is the absolute distance of the threshold from the maximal element in the linear extension.	
poverty_gap	Population mean of svr_rel	
wealth_gap	Population mean of wea_rel	
inequality	when the argument inequality is TRUE, the average value of the inequality index over the linear extensions (see Fattore and Arcagni, 2013). Function inequalty returns only this result.	

# References

Bubley R., Dyer M. (1999), Faster random generation of linear extensions, Discrete Math., 201, 81-88.

Fattore M., Arcagni A. (2013), Measuring multidimensional polarization with ordinal data, SIS 2013 Statistical Conference, BES-M3.1 - The BES and the challenges of constructing composite indicators dealing with equity and sustainability

```
profiles <- var2prof(varlen = c(3, 2, 2))
threshold <- c("311", "112")
res <- evaluation(profiles, threshold, maxint = 10^5)
summary(res)
plot(res)</pre>
```

# Description

The function FOD performs the Fuzzy First Order Dominance analysis described in Fattore and Arcagni (forthcoming).

#### Usage

```
FFOD(profiles, ...)
## S3 method for class 'wprof'
FFOD(profiles,
    distributions = as.data.frame(profiles$freq),
    lambda = do.call(
        getlambda, as.list(names(profiles$profiles))
    ),
    alpha = NULL, ...
)
```

# Arguments

profiles	an object of class wprof.
distributions	a data.frame of frequencies/weights where the columns correspond to the dif- ferent distributions and the rows to the profiles. The profiles in the rows have to be ordered as in profiles.
lambda	object of class incidence representing the partial order of the relative impor- tance of the indicators. By default, the lambda poset is an antichain ( i.e. all the indicators are considered equi-important).
alpha	vector of values to cut the mintr.delta matrix to generate the posets in the list covers. Default is NULL posets are generated for each different value in matrix mintr.delta. See section 'Value' below.
	any of above.

#### Details

The function requires the set of profiles, through the object profiles of class wprof, and the corresponding frequencies, which can be defined by the argument distributions of class data.frame.

Notice that a warning is provided if the rownames of the distributions do not match the rownamens of the profiles.

Through poset lambda, it is possible to provide (ordinal) information on the relative importance of the indicators in the multi-indicator system.

# FOD

# Value

An object of class FODposet containing:

delta	matrix of the overall dominance degrees.
mintr.delta	matrix of the min-transitive closure of matrix delta.
global.approx global.approx.o	L1 distance between delta and mintr.delta, divided by the L1 norm of delta.
Brondruppi ov	L1 distance between delta and mintr.delta, divided by the L1 norm of delta after removing its diagonal.
cell.approx	matrix of absolute differences between the elements of delta and the elements of mintr.delta.
posets.ind	data.frame with indicators describing the partial orders obtained as alpha-cuts of the min-transitive closure mintr.delta. For each poset, the data frame provides: its cardinality, the number of comparabilities, the number of incomparabilities and their ratio (ci.ratio).
eqv.classes	list of boolean matrices specifying, for each alpha-cut, the equivalence classes of the input distributions. Equivalence classes are reported by rows and the initial distributions by columns. If element ij of the matrix is TRUE, then distribution j belongs to the i-th equivalence class.
covers	list of objects of class cover comprising the cover matrices of the poset gener- ated by each alpha-cut of mintr.delta.

#### Author(s)

Fattore M., Arcagni A.

#### References

Fattore M., Arcagni A. (forthcoming), F-FOD: Fuzzy First Order Dominance analysis and populations ranking over ordinal multi-indicator systems.

```
v1 <- as.ordered(c("a", "b", "c", "d"))
v2 <- 1:3
prof <- var2prof(varmod = list(v1 = as.ordered(c("a", "b", "c", "d")), v2 = 1:3))
np <- nrow(prof$profiles)
k <- 10 # number of populations
set.seed(0)
populations <- as.data.frame(lapply(1:k, function(x) round(runif(np)*100)))
rownames(populations) <- rownames(prof$profiles)
names(populations) <- paste0("P", 1:k)
prof
populations
res <- FFOD(profiles = prof, distributions = populations)
res
```

gen.downset

# Description

The function returns the antichain generating the input downset Q, given the incidence matrix z of the poset.

#### Usage

gen.downset(z, Q = 1)

#### Arguments

Z	an incidence matrix.
Q	a vector (boolean, numeric indexing elements, or character with elements' names) identifying the input downset.

#### Value

A boolean vector.

#### See Also

gen.upset

```
lv <- c(2, 3, 2)
prof <- var2prof(varlen = lv)
z <- getzeta(prof)
down <- c("111", "211", "112", "212")
gen <- gen.downset(z, down)
plot(z, lwd = 1 + (rownames(prof$profiles)%in%down), col = 1 + gen,
sub = "bold = the downset, red = the antichain generating the downset")</pre>
```

gen.upset

# Description

The function returns the antichain generating the input upset Q, given the incidence matrix z of the poset.

#### Usage

gen.upset(z, Q = 1)

#### Arguments

Z	an incidence matrix.
Q	a vector (boolean, numeric indexing elements, or character with elements' names) identifying the input upset.

#### Value

A boolean vector.

#### See Also

gen.downset

```
lv <- c(2, 3, 2)
prof <- var2prof(varlen = lv)
z <- getzeta(prof)
up <- c("221", "131", "231", "222", "132", "232")
gen <- gen.upset(z, up)
plot(z, lwd = 1 + (rownames(prof$profiles)%in%up), col = 1 + gen,</pre>
```

getlambda

*Object constructor for the incidence matrix representing a partial order on variables.* 

# Description

The function creates an object of class incidence representing a partial order on the set of variables.

#### Usage

getlambda(...)

#### Arguments

... Cover relations between variable pairs.

#### Details

Cover relations between pair of ariables are defined by the names of the two variables and the symbols < and >. For instance, if variable A is covered by variable B, write the cover relation as A < B or B > A. If a variable is not comparable to the others, write the name of the variable alone.

#### Value

an object of class incidence.

# Author(s)

Alberto Arcagni

# See Also

plot.cover

```
Lambda <- getlambda(BOTTOM < A, B > BOTTOM, INCOMP)
plot(Lambda)
```

getzeta

# Description

The function computes the incidence matrix from the set of input profiles y. The output is a boolean matrix of S3 class incidence.

#### Usage

```
getzeta(y)
## S3 method for class 'wprof'
getzeta(y)
```

## Arguments

у

the set of profiles, an object of S3 class wprof.

# Examples

prf <- var2prof(varlen = c(2, 3))
getzeta(prf)</pre>

heights

Heights

# Description

The function computes the vector of heights of poset elements.

# Usage

heights(z)

# Arguments

z

an object of class cover, incidence or poset.

#### Examples

```
vl <- c(3, 2, 4)
prof <- var2prof(varlen = vl)
Z <- getzeta(prof)</pre>
```

heights(Z)

#### Description

Given a partial order (arguments profiles and/or zeta) and a selected threshold, the function computes the identification function, as a S3 class object parsec. The identification function is computed by uniform sampling of the linear extensions of the input poset, through a C implementation of the Bubley - Dyer (1999) algorithm. idn is a simplified and faster version of evaluation, computing just the identification function.

# Usage

```
idn(
    profiles = NULL,
    threshold,
    error = 10^{(-3)},
    zeta = getzeta(profiles),
    weights = {
        if (!is.null(profiles))
            profiles$freq
        else rep(1, nrow(zeta))
    },
    linext = lingen(zeta),
    nit = floor({
        n <- nrow(zeta)</pre>
        n^{5} * log(n) + n^{4} * log(error^{-1}))
    }),
    maxint = 2^{31} - 1
)
```

# Arguments

profiles	an object of S3 class wprof.
threshold	a vector identifying the threshold. It can be a vector of indexes (numeric), a vector of poset element names (character) or a boolean vector of length equal to the number of elements.
error	the "distance" from uniformity in the sampling distribution of linear extensions.
zeta	the incidence matrix of the poset. An object of S3 class incidence. By default, extracted from profiles.
weights	weights assigned to profiles. If the argument profiles is not NULL, weights are by default set equal to profile frequencies, otherwise they are set equal to 1.
linext	the linear extension initializing the sampling algorithm. By default, it is gener- ated by lingen(zeta). Alternatively, it can be provided by the user through a vector of elements positions.

# idn

idn

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nit	Number of iterations in the Bubley-Dyer algorithm, by default evaluated using a formula of Karzanov and Khachiyan based on the number of poset elements and the argument error (see Bubley and Dyer, 1999).
maxint	Maximum integer. By default the maximum integer obtainable in a 32bit system. This argument is used to group iterations and run the compiled C code more times, so as to avoid memory indexing problems. User can set a lower value to maxint in case of lower RAM availability.

# Value

profiles	an object of S3 class wprof reporting poset profiles and their associated frequencies (number of statistical units in each profile).	
number_of_profi	iles	
	number of profiles.	
number_of_varia	ables	
	number of variables.	
incidence	S3 class incidence, incidence matrix of the poset.	
cover	S3 class cover, cover matrix of the poset.	
threshold	boolean vector specifying whether a profile belongs to the threshold.	
number_of_itera	ations	
	number of iterations performed by the Bubley Dyer algorithm.	
rank_dist	matrix reporting by rows the relative frequency distribution of the poverty ranks of each profile, over the set of sampled linear extensions.	
thr_dist	vector reporting the relative frequency a profile is used as threshold in the sam- pled linear extensions. This result is useful for a posteriori valuation of the poset threshold.	
prof_w	vector of weights assigned to each profile.	
edges_weights	matrix of distances between profiles, used to evaluate the measures of gap.	
idn_f	vector reporting the identification function, computed as the fraction of sample linear extensions where a profile is in the downset of the threshold.	
svr_abs	NA use evaluation to obtain this result.	
svr_rel	NA use evaluation to obtain this result.	
wea_abs	NA use evaluation to obtain this result.	
wea_rel	NA use evaluation to obtain this result.	
poverty_gap	NA use evaluation to obtain this result.	
wealth_gap	NA use evaluation to obtain this result.	
inequality	NA use evaluation to obtain this result.	

# References

Bubley R., Dyer M. (1999), Faster random generation of linear extensions, Discrete Math., 201, 81-88.

Fattore M., Arcagni A. (2013), Measuring multidimensional polarization with ordinal data, SIS 2013 Statistical Conference, BES-M3.1 - The BES and the challenges of constructing composite indicators dealing with equity and sustainability

#### Examples

```
profiles <- var2prof(varlen = c(3, 2, 4))
threshold <- c("311", "112")
res <- idn(profiles, threshold, maxint = 10^5)
summary(res)
plot(res)</pre>
```

incidence2cover *incidence2cover* 

# Description

The function computes the cover matrix associated to the input incidence matrix (i.e. the cover matrix whose transitive closure is the input incidence matrix).

#### Usage

incidence2cover(z)

#### Arguments

z

an incidence matrix, an object of class incidence.

#### Value

Cover matrix, an object of class cover.

#### See Also

cover2incidence

incomparability Incomparability between profiles

# Description

The function computes the set of pairwise incomparabilities between poset elements.

#### Usage

incomp(z)

#### Arguments

z an incidence matrix.

#### 24

#### is.downset

# Value

A boolean matrix whose element ij is TRUE when profiles i and j are incomparable.

# See Also

getzeta

# Examples

```
vl <- c(2, 2, 2)
pr <- var2prof(varlen = vl)
Z <- getzeta(pr)
incomp(Z)</pre>
```

is.downset

is.downset

# Description

The function checks whether the input set of poset elements Q is a downset of the poset represented by the incidence matrix z.

# Usage

is.downset(z, Q = 1)

#### Arguments

Z	incidence matrix
Q	vector identifying the input set of profiles.

```
z <- getzeta(var2prof(varlen = c(2, 2, 2)))</pre>
```

```
plot(z, col = 1 + c(1, 1, 0, 0, 1, 0, 0, 0) + c(0, 0, 0, 2, 0, 0, 2, 2), lwd = 2)
```

```
Q <- c(4, 7, 8)
rownames(z)[Q]
is.downset(z, Q)
Q <- c("211", "112", "111")
is.downset(z, Q)</pre>
```

is.linext

#### Description

The function checks whether the input argument order is a linear extension of the poset represented by the incidence matrix z.

# Usage

is.linext(order, z)

# Arguments

order	indexes of the poset elements (as rows and columns of z matrix) specifying the
	candidate linear order.
Z	incidence matrix.

#### Examples

```
Z <- getzeta(var2prof(varlen = c(3, 3)))
ranks <- c(1, 4, 2, 3, 5, 7, 6, 8, 9)
names(ranks) <- rownames(Z)
ranks
is.linext(order = ranks, z = Z)</pre>
```

is.partialorder is.partialorder

# Description

The function checks whether the input boolean square matrix m represents a partial order.

# Usage

```
is.partialorder(m)
```

#### Arguments

m a boolean square matrix..

#### See Also

transitivity, binary, reflexivity, antisymmetry, is.preorder, validate.partialorder.incidence

# is.preorder

# Examples

```
M <- c(TRUE, FALSE, FALSE, FALSE, TRUE, TRUE, TRUE, FALSE, TRUE, FALSE, TRUE, FALSE, TRUE, TRUE, TRUE, TRUE, TRUE)
M <- matrix(M, 4, 4)
rownames(M) <- colnames(M) <- LETTERS[1:4]
is.partialorder(M)</pre>
```

is.preorder *is.preorder* 

# Description

The function checks whether the input boolean square matrix m represents a preorder.

#### Usage

is.preorder(m)

#### Arguments

m a boolean square matrix.

# See Also

transitivity, binary, reflexivity, antisymmetry, is.partialorder, validate.partialorder.incidence

# Examples

```
M <- c(TRUE, FALSE, FALSE, FALSE, TRUE, TRUE, TRUE, FALSE, TRUE, FALSE, TRUE, FALSE, TRUE, TRUE, TRUE, TRUE)
M <- matrix(M, 4, 4)
rownames(M) <- colnames(M) <- LETTERS[1:4]</pre>
```

is.preorder(M)

is.upset

#### Description

The function checks whether the input set of elements Q is an upset of the poset represented by the incidence matrix z.

#### Usage

is.upset(z, Q = 1)

#### Arguments

Z	an incidence matrix.
Q	vector specifying the input set of poset elements.

#### Examples

```
z <- getzeta(var2prof(varlen = c(2, 2, 2)))
plot(z, col = 1 + c(1, 1, 0, 0, 1, 0, 0, 0) + c(0, 0, 0, 2, 0, 0, 2, 2), lwd = 2)
Q <- c(4, 7, 8)
rownames(z)[Q]
is.upset(z, Q)
Q <- c("211", "112", "111")
is.upset(z, Q)</pre>
```

latex

latex

#### Description

The function returns the LaTeX code to create a tikz figure representing the Hasse diagram drawn from a set of profiles (prof), an incidence matrix (Z) or a cover matrix (C). The code can be copied and pasted into a latex file. The latex source requires the tikz package,

#### Usage

```
latex(y, ...)
## S3 method for class 'wprof'
latex(y, label = "", caption = "", scale = c(1, 1), ...)
## S3 method for class 'incidence'
latex(y, label = "", caption = "", scale = c(1, 1), ...)
## S3 method for class 'cover'
latex(y, label = "", caption = "", scale = c(1, 1), ...)
```

# Arguments

У	an object of S3 class wprof, an object of S3 class incidence or an object of S3 class cover.
label	the label of the LaTeX figure.
caption	the caption of the LaTeX figure.
scale	a vector of two elements to control the scale of the X-axis and the scale of the Y-axis in the LaTeX output.
	any of above.

# Examples

```
prof <- var2prof(varlen = c(2, 3))
latex(prof, label="fg:hasse", caption="Hasse diagram", scale = c(2, 2))</pre>
```

LE

# Linear extensions

# Description

The function generates all of the linear extensions of the partial order defined by the incidence matrix Lambda.

# Usage

LE(Lambda)

# Arguments

Lambda incidence matrix.

# Value

a list of vectors representing all linear orders compatible with the Lambda incidence matrix.

# Author(s)

Alberto Arcagni

# See Also

getlambda

# Examples

Lambda <- getlambda(A < B, A < C, D < C) LE(Lambda)

LE2incidence

# Description

The function generates the incidence matrices of the lexicographic linear extensions of a profile poset, given the variables (argument varmod or varlen) and a list of complete orders on them (argument lst).

#### Usage

```
LE2incidence(
    lst,
    varmod = lapply(as.list(varlen), function(lst) 1:lst),
    varlen = sapply(varmod, length)
)
## Default S3 method:
LE2incidence(
    lst,
    varmod = lapply(as.list(varlen), function(lst) 1:lst),
    varlen = sapply(varmod, length)
)
## S3 method for class 'list'
LE2incidence(
    lst,
    varmod = lapply(as.list(varlen), function(x) 1:x),
    varlen = sapply(varmod, length)
)
```

#### Arguments

lst	a vector of characters, or a list of specifies the names of the variables in in- creasing order. See details.
varmod	list of variables and their grades. See details.
varlen	a vector with the number of grades of each variable. See details.

#### Details

Argument 1st is a list of chracter vectors. Each vector lists variable names in increasing order.

List varmod and vector varlen must be named so as to identify the variables they refer to. Profiles are generated as combinations of the variables' grades. The names of the profiles are the grades of the variables concatenated, after the variables order in varmod/varlen. See var2prof for more details about these arguments.

# Value

an object of S3 class incidence or a list of objects of S3 class incidence.

# Author(s)

Alberto Arcagni

# See Also

var2prof

# Examples

```
Lambda <- getlambda(A < B, C < D)
plot(Lambda)
lst <- LE(Lambda)
vl <- c(A = 2, B = 2, C = 2, D = 2)
lstZeta <- LE2incidence(lst, varlen = vl)
for (x in lstZeta)
    plot(x)</pre>
```

levels.incidence and levels.cover Levels of a poset

# Description

The methods return a vector associating each profile with the corresponding level. The behaviour of these methods for objects of classes incidence and cover is different from the behaviour of function levels for factors.

# Usage

```
## S3 method for class 'incidence'
levels(x)
## S3 method for class 'cover'
levels(x)
```

#### Arguments

x an object of class cover or incidence.

#### See Also

the function levels for objects of type factor

linzeta

## Examples

```
vl <- c(3, 2, 4)
prof <- var2prof(varlen = vl)
Z <- getzeta(prof)</pre>
```

levels(Z)

lingen

lingen

# Description

The function computes a vector of ranks, defining a linear extension of the poset represented by incidence matrix z.

# Usage

lingen(z)

# Arguments

Z

an incidence matrix.

# Examples

Z <- getzeta(var2prof(varlen = c(3, 3)))
lingen(Z)</pre>

linzeta

linzeta

# Description

The function computes the incidence matrix of the linear order defined by the rank vector lin. It returns an object of S3 class incidence.

## Usage

linzeta(lin)

# Arguments

lin a vector of elements' ranks.

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#### maximal

#### Examples

```
ranks <- c(5, 3, 4, 2, 1)
names(ranks) <- LETTERS[1:5]
linzeta(ranks)
plot(linzeta(ranks))</pre>
```

maximal

Maximal elements of a poset.

# Description

The function returns a boolean vector identifying the maximal elements of the poset.

#### Usage

maximal(z)

# Arguments

z

an object of class cover or incidence.

#### Examples

vl <- c(3, 2, 4)
prof <- var2prof(varlen = vl)
Z <- getzeta(prof)</pre>

maximal(Z)

merge.wprof Merge two sets of profiles.

# Description

Method of the function merge of package base to merge two objects of class wprof generated through functions var2prof or pop2prof.

## Usage

```
## S3 method for class 'wprof'
merge(x, y, support = FALSE, FUN = "+", all = TRUE, ...)
```

#### Arguments

х, у	objects of class wprof to be coerced to one.
support	boolean variables specifying whether y is the support of x (FALSE by default).
FUN	function to be applied to the profiles' frequencies (by default, $FUN = sum$ ). It is ignored if support is TRUE.
all	same argument of function merge, by default set to TRUE, to get all possible profiles. If a profile is not observed in the data, its frequency is set to 0.
	additional arguments to be passed to method merge.data.frame of the package base.

#### Details

Objects of class wprof are composed of a data.frame of profiles and a vector of frequencies. This method applies method merge.data.frame to the profiles and applies function FUN to the frequencies.

If support is TRUE, function merge.data.frame is not used and the output corresponds to the object y, but with its frequencies modified. These are set equal to the frequencies of the corresponding profiles in x, or to 0 for profiles not contained in x.

#### Author(s)

Arcagni A.

#### See Also

merge, var2prof, pop2prof

```
n <- 5
v1 <- as.ordered(c("a", "b", "c", "d"))
v2 <- 1:3
set.seed(0)
pop <- data.frame(
v1 = sample(v1, n, replace = TRUE),
v2 = sample(v2, n, replace = TRUE)
)
survey_weights <- round(runif(5)*10)
prof1 <- pop2prof(pop, weights = survey_weights)
prof2 <- var2prof(varmod = list(v1 = as.ordered(c("a", "b", "c", "d")), v2 = 1:3))
# prof2 is the support of prof1
merge(prof1, prof2, support = TRUE)
# union between the two sets of profiles and their frequencies are added
merge(prof1, prof2)
```

#### minimal

```
# intersection of the sets of profiles with the assumption
# that the minimum number of observations is shared
# between the two distributions
merge(prof1, prof2, all = FALSE, FUN = min)
prof2$freq <- prof2$freq*10</pre>
```

```
# to remove from prof2 the observations in prof1
distribution <- merge(prof2, prof1, FUN = "-"); distribution</pre>
```

minimal

#### Minimal elements of a poset

# Description

The function returns a boolean vector identifying the minimal elements of the poset.

# Usage

minimal(z)

#### Arguments

z an object of class cover or incidence.

#### Examples

```
vl <- c(3, 2, 4)
prof <- var2prof(varlen = vl)
Z <- getzeta(prof)</pre>
```

minimal(Z)

mrg

Merge posets

# Description

The function merges posets defined through a list of incidence matrices or a list of complete orders between the variables (argument lst). In the second case the variables must be defined (argument varmod or varlen).

# Usage

```
mrg(
    lst,
    varmod = lapply(as.list(varlen), function(x) 1:x),
    varlen = sapply(varmod, length)
)
## S3 method for class 'incidence'
mrg(lst, varmod = NULL, varlen = NULL)
## S3 method for class 'character'
mrg(
lst,
    varmod = lapply(as.list(varlen), function(x) 1:x),
    varlen = sapply(varmod, length)
)
```

#### Arguments

lst	a list of incidence matrices (class incidence) or list of vectors of characters. See details.
varmod	list of variables and their grades. See details.
varlen	a vector of number of grades of each variable. See details.

#### Details

For efficiency reasons, the argument lst can be also a list of vectors of characters. In this case, each vector lists the names of the variables in increasing order.

The list varmod and the vector varlen must be named, so as to identify the variables they refer to. The profiles are generated by the combinations of the variables grades. The names of the profiles are the grades of the variables concatenated, according to variables order in varmod/varlen. See var2prof for more details about these arguments.

#### Value

an object of S3 class incidence.

#### Author(s)

Alberto Arcagni

#### See Also

var2prof, LE2incidence

#### Examples

```
# Example with lst as list of incidence matrices
Lambda <- getlambda(A < B, C < D)
plot(Lambda)
lst <- LE(Lambda)</pre>
```

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## MRP

```
vl <- c(A = 2, B = 2, C = 2, D = 2)
lstZeta <- LE2incidence(lst, varlen = vl)
for (x in lstZeta)
    plot(x)
mrg(lstZeta)
# Example with lst as list of characters
Lambda <- getlambda(A < B, C < D)
lst <- LE(Lambda)
vl <- c(A = 2, B = 2, C = 2, D = 2)
Zeta <- mrg(lst, varlen = vl)
plot(Zeta)</pre>
```

MRP

Mutial ranking probability matrix

#### Description

Function to evaluate Mutial Ranking Probability (MRP) matrix based on netrankr package.

## Usage

```
MRP(Z, method = c("exact", "mcmc", "approx"), error = 10^(-3), nit = NULL)
## S3 method for class 'incidence'
MRP(Z, method = c("exact", "mcmc", "approx"), error = 10^(-3), nit = NULL)
```

#### Arguments

Z	an incidence matrix, an object of class incidence.
method	a string to choose the method applied to evaluate the MRP matrix. The default value is "exact". See section 'Details' below.
error	considered only if mcmc method is selected. The "distance" from uniformity in the sampling distribution of linear extensions.
nit	considered only if mcmc method is selected. Number of ITerations in the Bubley- Dyer algorithm, by default evaluated indicated in Bubley and Dyer (1999) de- pending on the value of error.

#### Details

Package netrankr provides three functions to evaluate MRP matrix. Note that MRP matrix definition in netrankr is a little different from the one used in Fattore and Arcagni (2018), therefore this function unifies the results to the second definition.

Parameter method allows the selection of which function of package netrankr to use: "exact" runs the function exact\_rank\_prob that provides the exact results, "mcmc" the function mcmc\_rank\_prob that provide the estimated results through the Bubley Dyer algorithm and "approx" runs the function approx\_rank\_relative that provide the Bruggemann and Carlsen (2011) approximated results. For small posets it is possible to evaluate the exact MRP matrix, for larger posets it is necessary to use the appoximated results.

#### Value

An object of class matrix representing the MRP matrix. Dimensions names are equal to incidence matrix ones.

#### References

Bruggemann R., Carlsen L., (2011). An improved estimation of averaged ranks of partial orders. MATCH Commun. Math. Comput. Chem., 65(2):383-414.

Bubley R., Dyer M. (1999), Faster random generation of linear extensions, Discrete Math., 201, 81-88.

Fattore M., Arcagni A. (2018). Using mutual ranking probabilities for dimensionality reduction and ranking extraction in multidimensional systems of ordinal variables. Advances in Statistical Modelling of Ordinal Data, 117.

## See Also

exact\_rank\_prob, mcmc\_rank\_prob, approx\_rank\_relative

## Examples

L <- getlambda(A < B, C < B, B < D) MRP(L)

MRPlex	Mutual ranking probabilities on the lexicographic linear extensi	ons
	set	

#### Description

The function returns the mutual ranking probabilities matrix evaluated considering only the lexicographic linear extensions. Results are obtained by exact formula.

#### Usage

```
MRPlex(profiles, selection = NULL)
```

#### Arguments

profiles	an object of S3 class wprof.
selection	a vector of string indicating a subset of profiles to evaluate the mutual rank- ing probabilities. If NULL the mutual ranking probabilities are evaluated for all profiles.

## Value

The MRP matrix of the selected profiles.

#### obsprof

#### Examples

```
prf <- var2prof(varlen = c(2, 2, 2))
MRPlex(prf)</pre>
```

obsprof

Remove unobserved profiles.

#### Description

The function removes, from the set of possible profiles prof derived from the multi-indicator system, those unobserved in the input dataset (i.e. profiles with associated frequency equal to zero). It returns an object of class S3 wprof comprising the observed profiles and their frequencies.

#### Usage

obsprof(prof)
## S3 method for class 'wprof'
obsprof(prof)

# Arguments prof

object of S3 class wprof.

#### Examples

prf <- var2prof(varlen = c(3, 3, 3))
prf\$freq <- sample(c(0, 1), 3\*3\*3, replace = TRUE)
prf <- obsprof(prf)
plot(prf, shape = "equispaced")</pre>

parsec2igraph Converting a partial order to an object of the package igraph.

#### Description

The function turns a cover matrix to an **igraph** object, so as to allow using the graphical power of **igraph** to plot Hasse diagrams. Objects of class cover are boolean matrices where element ij is equal to 1 if element i *is covered by* element j. This makes the cover matrix the transpose of the adjacency matrix of a graph, describing the cover relation in **igraph**.

## Usage

```
parsec2igraph(p, ...)
## S3 method for class 'cover'
parsec2igraph(p, ...)
## S3 method for class 'incidence'
parsec2igraph(p, ...)
```

#### Arguments

р	an object of class cover or incidence.
	additional arguments of the function vertices.

### Value

The function returns an object of class igraph, representing the directed graph defined by the cover relation.

The function adds to the graph a layout generated through function vertices, so as to plot the graph according to the conventions used for Hasse diagrams.

## Author(s)

Arcagni, A.

## References

Csardi G, Nepusz T: The igraph software package for complex network research, InterJournal, Complex Systems 1695. 2006. http://igraph.org

## See Also

igraph, vertices

## Examples

```
example(merge.wprof)
poset <- getzeta(distribution)
incidence2cover(poset)

G <- parsec2igraph(poset, noise = TRUE)
get.adjacency(G)
# tkplot(G, vertex.size = distribution$freq, vertex.color = "white")

G <- parsec2igraph(poset, noise = 10)
# tkplot(G, vertex.size = distribution$freq, vertex.color = "white")</pre>
```

plot.average\_ranks Method of function plot for objects of class average\_ranks

## Description

From the output of the function average\_ranks, the function plots the average rank and the associated rank interval, for each element of the poset.

plot.average\_ranks

## Usage

```
## S3 method for class 'average_ranks'
plot(x,
    range.first = TRUE, range.col = "black", range.lty = 1,
    range.lwd = 1, type = "p", ylim = c(nrow(x), 1),
    xlab = "", ylab = "Average rank", pch = c(16, 3, 3),
    col = "black", cex = c(1, 1, 1), ...
)
```

## Arguments

х	An object of class average_ranks.
range.first	A boolean attribute to specify whether the interval is plotted in background (TRUE) or in foreground (FALSE).
range.col	Color of the interval.
range.lty	The line type to represent the range; the values are the same of the attribute lty in the plot.default function.
range.lwd	Width of the lines representing the range.
type	Attribute of the function plot.default, here "p" by default.
ylim	Attribute of function plot.default, here c(max(x\$sup), 1) by default (this way, the Y-axis is reversed, so that rank 1 corresponds to "best").
xlab	Attribute of the function plot.default, here "" by default.
ylab	Attribute of the function plot.default, here "Average rank" by default.
pch	Attribute of the function plot.default, here c(16, 3, 3) by default. This method uses the matplot function to plot the average ranks and their range. The first value refers to the point character of the average rank, the other two to the point characters of the range.
col	Attribute of the function plot.default, here "black" by default. The average ranks and their ranges are of the same color, but similarly to pch, users can provide a vector of different colors.
cex	Attribute of the function plot.default, here c(1, 1, 1) by default.
	Other arguments of the function plot.default.

## See Also

average\_ranks, plot.default, matplot

#### Examples

```
profiles <- var2prof(varlen = c(3, 2, 4))
Z <- getzeta(profiles)
res <- average_ranks(Z)
plot(res)</pre>
```

plot.cover

#### Description

plot methods to draw Hasse diagrams, for objects of S3 classes wprof, incidence, cover,

#### Usage

## Arguments

x	an object of S3 class wprof, an object of S3 class incidence or an object of S3 class cover.
shape	shape of the Hasse diagram. See vertices.
,	
noise	jittering in the shape of the Hasse diagram. See vertices.
pch	graphical parameter. See plot.default.
cex	graphical parameter. See plot.default.
bg	graphical parameter. See plot.default.
	further optional graphical parameters. See plot.default.

## Examples

prf <- var2prof(varlen = c(5, 5, 5))
prf\$freq <- sample(c(rep(0, 20), 1, 2, 3), 5\*5\*5, replace = TRUE)
prf <- obsprof(prf)
z <- getzeta(prf)
plot(z, shape = "equispaced", col = prf\$freq, lwd = 2)</pre>

plot.parsec

# Description

Several representations of the results provided by the evaluation function.

# Usage

```
## S3 method for class 'parsec'
plot(
    x,
    which = c("Hasse", "threshold", "identification", "rank", "gap"),
    ask = dev.interactive(),
    shape = c("square", "circle", "equispaced"),
    noise = FALSE,
    ...
)
```

## Arguments

х	an object of S3 class parsec, output of the evaluation function.
which	the names of the graphs to be plotted (all, by default); the user can choose among
	• Hasse, the Hasse diagram of the poset, see plot.cover for details,
	• threshold, the relative frequencies of the times a profile is used as threshold in the sampled linear extensions.
	• rank, barplot providing the rank distribution of each profile (X-axis). The heights of the blocks represent relative frequencies (the sum of the heights over profiles is equal to 1) and the color represents the rank: white for rank one, black for the highest rank and a gray scale for intermediate ranks.
	• gap, a unified representation of the relative (e.g. poverty) gap and of the relative (e.g. wealth) gap. The horizontal lines represent the average (e.g. poverty) gap and the average (e.g. wealth gap). The darker vertical dashed lines represent the threshold profiles.
ask	boolean value indicating whether the system has to ask users before changing the plot.
shape	the shape of the Hasse diagram, see plot.cover for details.
noise	jittering in the shape of the Hasse diagram. See vertices.
	further arguments for the plot.cover function.

#### See Also

evaluation, plot.cover

#### Examples

```
profiles <- var2prof(varlen = c(3, 2, 4))
threshold <- c("311", "112")
res <- evaluation(profiles, threshold, nit = 10^3)
plot(res)</pre>
```

plot.rank\_stability *Plot outputs of PARSEC function* rank\_stability.

## Description

The function generates four plots, to reproduce the sequence of the average ranks and of the positions of the elements, in the rankings associated to the alpha-cut posets.

Rankings and average ranks have to be evaluted with the function rank\_stability.

First and third plots show the sequence of average ranks, second and fourth show the sequence of rankings. Sequences in first and second plots are shown against the sequence of alpha-cuts, in third and fourth plots as a function of alpha values.

#### Usage

```
## S3 method for class 'rank_stability'
plot(x,
    which = 1:4, legend = TRUE, legend.x = "bottomleft",
    legend.y = NULL, legend.bg = "white", grid = TRUE,
    grid.lty = 2, grid.col = rgb(0, 0, 0, 1/7),
    grid.lwd = 1, y_axis = "reversed", ask = dev.interactive(),
    type = "l", col = gray(1:ncol(x$ranking)/ncol(x$ranking)/1.3),
    lwd = 3, lty = 1, ...
```

)

#### Arguments

х	object of class rank_stability generated by function rank_stability.	
which	select a subset of the numbers 1:4, to specify the desired plots. See caption below (and the 'Details').	
legend	boolean argument to choose whether to show the legend in the plots.	
<pre>legend.x, legend.y, legend.bg</pre>		
	arguments x, y and bg of the function legend defining the coordinates and the backgroud color of the legend.	
grid	boolean argument to choose whether to show the grid in the plots.	
grid.lty, grid.col, grid.lwd		
	arguments defining the line type, color and width of the grid.	
y_axis	if it is set equal to "reversed" plots show the y axis reversed.	

## pop2prof

ask	boolean argument indicating whether the system has to ask users before chang- ing plots.
type	1-character string giving the desired type of plot. See plot.default for details.
col	vector of colors. See matplot for details.
lwd	vector of line widths. See matplot for details.
lty	vector of line types. See matplot for details.
	other arguments of function matplot.

## See Also

rank\_stability, legend, plot.default, matplot

## Examples

```
v1 <- as.ordered(c("a", "b", "c", "d"))
v2 <- 1:3
prof <- var2prof(varmod = list(v1 = as.ordered(c("a", "b", "c", "d")), v2 = 1:3))
np <- nrow(prof$profiles)
k <- 10 # number of populations
set.seed(0)
populations <- as.data.frame(lapply(1:k, function(x) round(runif(np)*100)))
rownames(populations) <- rownames(prof$profiles)
names(populations) <- paste0("P", 1:k)
x <- FFOD(profiles = prof, distributions = populations)
res <- rank_stability(x)
plot(res)
```

pop2prof

Population to profiles

## Description

Extract the observed profiles and the corresponding frequencies, out of the statistical population.

#### Usage

```
pop2prof(
   y,
   labtype = c("profiles", "progressive", "rownames"),
   sep = "",
   weights = rep(1, nrow(y))
)
```

#### Arguments

У	a dataset, used to count profile frequencies. See details.
labtype	users can choose the type of labels to assign to profiles. See details.
sep	variables separator in the profiles labels.
weights	a vector of length equal to the number of observations in y, representing the survey weights of each observation.

#### Details

y is a data.frame of observations on the ordinal or numeric variables. The partial order must be defined within the object type, so as to build the incidence matrix of the order relation (see getzeta).

The function extracts variables and their observed modalities from the population; it builds all possible profiles and assigns to them the corresponding frequency. If some modalities are not observed in the population, they will not be used to build the profiles. If one is interested in the set of all possible profiles from a given set of variables, function var2prof is to be used.

Users can choose the label type to assign to profiles. Accepted types are: profiles the variabiles modalities, progressive a progressive numeration, rownames the rownames in the dataset.

## Value

The function returns a S3 class object wprof, "weighted profiles", containing the data.frame named profiles and the frequency vector freq.

#### See Also

var2prof, getzeta

## Examples

```
n <- 5
v1 <- as.ordered(c("a", "b", "c", "d"))
v2 <- 1:3
pop <- data.frame(
    v1 = sample(v1, n, replace = TRUE),
    v2 = sample(v2, n, replace = TRUE)
)
pop2prof(pop)</pre>
```

popelem

popelem

#### Description

The function identifies in a matrix y, profiles in prof. For each row of matrix y, the function returns the location of the corresponding profile in object prof.

## proFreq

## Usage

```
popelem(prof, ...)
## S3 method for class 'wprof'
popelem(prof, y, ...)
```

#### Arguments

prof	an object of S3 class wprof.
У	a matrix or data.frame representing a set of observations with variables (the same contained in prof) by columns.
	any of the above.

## Examples

```
vl <- c(2, 3, 2)
prf <- var2prof(varlen = vl)
pop <- matrix(c(2, 1, 1, 1, 2, 1, 2, 3, 1), 3, 3)
rownames(pop) <- LETTERS[1:3]
v <- popelem(prof = prf, y = pop)
v
prf$profiles[v,]</pre>
```

proFreq

Observed profile frequencies

## Description

The function computes profile frequencies, by counting the number of times a profile appears in the population.

## Usage

```
proFreq(profiles, population)
```

#### Arguments

profiles	an object of S3 class wprof.
population	a matrix or data.frame representing a set of observations with variables (the
	same contained in prof) by columns.

# Value

An object of class wprof with the same profiles of the argument but with different frequencies.

## Author(s)

Alberto Arcagni

## See Also

popelem

# Examples

```
vl <- c(2, 3, 2)
prf <- var2prof(varlen = vl)
pop <- matrix(c(2, 1, 1, 1, 2, 1, 2, 3, 1), 3, 3)
rownames(pop) <- LETTERS[1:3]</pre>
```

```
proFreq(profiles = prf, population = pop)
```

rank\_stability Rank stability analysis in posetic FOD

## Description

The function computes the average ranks and the positions in the ranking of the elements of the alpha-cuts.

#### Usage

```
rank_stability(x, ...)
## S3 method for class 'FODposet'
rank_stability(x,
    selection = 1:length(x$covers),
    coverage_probability = 0.9,
    error = 10^(-5), ...
)
```

#### Arguments

x	object of class FODposet generated by function FOD.
selection	numeric vector or a vector of names to select the cover matrices in argument x.
coverage_probal	bility least coverage probability of the rank intervals with extremes lower_ranks and upper_ranks.
error	the "distance" from uniformity in the sampling distribution of linear extensions used to evaluate the average ranks. See idn for details.
	any of above.

## reflexivity

## Value

alpha	vector of the alpha values defining the alpha-cuts.
average_ranks	data frame of average ranks of the poset elements (by columns) at different alpha values (by rows).
lower_ranks	data frame of the lower bounds of the rank interval, of each poset element (by columns) at different alpha values (by rows).
upper_ranks	data frame of the upper bounds of the rank interval, of each poset element (by columns) at different alpha values (by rows).
ranking	data frame of the positions of poset elements (by columns), in the ranking ex- tracted from the posets associated to alpha-cuts (by rows).
resolution	number of elements of the posets associated to the alpha-cuts.

## Author(s)

Fattore M., Arcagni A.

#### See Also

FFOD, idn

## Examples

```
v1 <- as.ordered(c("a", "b", "c", "d"))
v2 <- 1:3
prof <- var2prof(varmod = list(v1 = as.ordered(c("a", "b", "c", "d")), v2 = 1:3))
np <- nrow(prof$profiles)
k <- 10 # number of populations
set.seed(0)
populations <- as.data.frame(lapply(1:k, function(x) round(runif(np)*100)))
rownames(populations) <- rownames(prof$profiles)
names(populations) <- paste0("P", 1:k)
x <- FFOD(profiles = prof, distributions = populations)
res <- rank_stability(x)
res
```

reflexivity reflexivity

## Description

The function checks whether the input boolean square matrix m represents a reflexive binary relation.

## Usage

reflexivity(m)

#### Arguments

m

a boolean square matrix.

## See Also

transitivity, binary, antisymmetry, is.preorder, is.partialorder,

validate.partialorder.incidence

#### Examples

```
M <- c(TRUE, FALSE, FALSE, FALSE, TRUE, TRUE, TRUE, FALSE, TRUE, FALSE, TRUE, FALSE, TRUE, TRUE, TRUE, TRUE)
M <- matrix(M, 4, 4)
rownames(M) <- colnames(M) <- LETTERS[1:4]</pre>
```

reflexivity(M)

rmProfiles

Removing profiles

## Description

Function to remove profiles from an object of class wprof.

## Usage

```
rmProfiles(y, ...)
## S3 method for class 'wprof'
rmProfiles(y, v, ...)
```

#### Arguments

У	object of class wprof.
v	a vector pointing to the profiles to be removed. The vector can be of type:
	• numeric whose components refer to the positions of profiles in y;
	• logical of the same length as the number of profiles in y;
	<ul> <li>character, referring to profile names in y.</li> </ul>
	any of the above.

## Value

The function returns an wprof object equal to y but without the profiles in v.

#### summary.cover

## Examples

```
vl <- c(3, 3, 3)
prof <- var2prof(varlen = vl)
rownames(prof$profiles)
prof <- rmProfiles(prof, c("123", "321"))
plot(prof)</pre>
```

summary.cover

## Summary method for cover and incidence objects.

# Description

The function computes a summary of cover and incidence S3 objects. Currently, the function returns just the number of profiles and the number of comparabilities.

#### Usage

```
## S3 method for class 'cover'
summary(object, ...)
## S3 method for class 'incidence'
summary(object, ...)
```

## Arguments

object	a cover matrix or an incidence matrix.
	added for consistency with the generic method.

## Examples

```
vl <- c(2, 3, 3)
prf <- var2prof(varlen = vl)
Z <- getzeta(prf)
summary(Z)
C <- incidence2cover(Z)
summary(C)</pre>
```

## Description

S3 method of function summary reporting main information for an object of class parsec, obtained from function evaluation. In particular, the function computes a table showing, for each profile:

- the variables' grades identifying the profile (if these are returned by evaluation.
- the assigned weight.
- whether or not it belongs to the threshold.
- the corresponding value of the identification function.
- the average poverty rank.
- the different gap measures (see evaluation for details).

If the number of profiles is higher than ten, the shown table gets cut, but the method returns a data.frame providing the complete output.

#### Usage

## S3 method for class 'parsec'
summary(object, ...)

#### Arguments

object	an object of S3 class parsec, output of the evaluation function.
	added for consistency with the generic method.

## See Also

evaluation

#### Examples

```
profiles <- var2prof(varlen = c(3, 2, 4))
threshold <- c("311", "112")
res <- evaluation(profiles, threshold, nit = 10^3)
sm <- summary(res)
summary(sm)</pre>
```

## Description

The function computes the transitive closure of a reflexive and antisymmetric binary relation.

## Usage

```
transitiveClosure(m)
```

#### Arguments

m

a generic square boolean matrix representing a reflexive and antisymmetric binary relation, an object of class cover or an object of class incidence.

## Value

Incidence matrix of the transitive closure of the input matrix m.

#### See Also

is.partialorder

## Examples

m <- c(1, 0, 0, 1, 0, 1, 0, 1, 0, 0, 1, 0, 1, 1, 0, 1)
m <- matrix(m, 4, 4)</pre>

transitiveClosure(m)

transitivity transitivity

#### Description

The function checks whether the boolean square matrix m representes a transitive binary relation.

#### Usage

```
transitivity(m)
```

#### Arguments

m a boolean square matrix.

upset

#### See Also

binary, reflexivity, antisymmetry, is.preorder, is.partialorder, validate.partialorder.incidence

## Examples

```
M <- c(TRUE, FALSE, FALSE, FALSE, TRUE, TRUE, TRUE, FALSE, TRUE, FALSE, TRUE, FALSE, TRUE, TRUE, TRUE, TRUE)
M <- matrix(M, 4, 4)
rownames(M) <- colnames(M) <- LETTERS[1:4]</pre>
```

```
transitivity(M)
```

upset

upset

## Description

The function computes a boolean vector specifying which poset elements belong to the downset generated by subposet Q.

#### Usage

```
upset(z, ...)
## S3 method for class 'cover'
upset(z, ...)
## S3 method for class 'incidence'
upset(z, Q = NULL, ...)
```

## Arguments

Z	a cover or an incidence matrix, of S3 classes cover or incidence, respectively.
Q	vector specifying a subposet of the poset represented by z.
	any of the above.

## Examples

z <- getzeta(var2prof(varlen = c(2, 2, 2)))</pre>

```
plot(z, col = 1 + c(1, 1, 0, 0, 1, 0, 0, 0) + c(0, 0, 0, 2, 0, 0, 2, 2), lwd = 2)
Q <- c(4, 7, 8)
rownames(z)[Q]
upset(z, Q)
Q <- c("211", "112", "111")
upset(z, Q)</pre>
```

validate.partialorder.incidence validate.partialorder.incidence

## Description

The function checks whether the boolean square matrix m represents a partial order. If yes, the function returns the same input matrix as a S3 class object incidence. Otherwise, the unfulfilled partial order properties of matrix m are returned.

#### Usage

validate.partialorder.incidence(m)

# Arguments m

a boolean square matrix.

#### See Also

transitivity, binary, reflexivity, antisymmetry, is.preorder, is.partialorder

#### Examples

```
M <- c(TRUE, FALSE, FALSE, FALSE, TRUE, TRUE, TRUE, FALSE, TRUE, FALSE, TRUE, FALSE, TRUE, TRUE, TRUE, TRUE)
M <- matrix(M, 4, 4)
rownames(M) <- colnames(M) <- LETTERS[1:4]</pre>
```

M <- validate.partialorder.incidence(M)</pre>

plot(M)

var2prof

Variables to profiles

#### Description

The function computes the list of all of the profiles from a list of input ordinal variables. See details for how to define variables.

#### Usage

```
var2prof(varmod = lapply(as.list(varlen), function(x) 1:x),
varlen = sapply(varmod, length), freq = NULL,
labtype = c("profiles", "progressive"), y=NULL)
```

#### Arguments

varmod	list of variables and their grades. See details.
varlen	a vector of number of grades of each variable. See details.
freq	profiles frequency distribution. By default, the frequencies are set equal to 1.
labtype	type of labels to assign to profiles. See details.
У	a matrix of observations, used to count profiles frequencies. See details.

#### Details

Variables can be defined through their names and grades, using a list as argument varmod. The names of the objects in the list are taken as variable names. The objects in the list must be ordered vectors or numeric vectors.

A faster way to define variables is through a vector with the number of grades of each variable, as argument varlen. This way, variables and their grades are assigned arbitrary names. In particular, grades are identified by their ranks in the variable definition.

The user can choose the type of label to assign to profiles. profiles is the combination of grades identifying the profiles. When the names of the grades are too long, it is suggested to choose progressive.

y is a matrix of observations on the ordinal variables (observations by rows and variables by columns). Variables must be ordered as defined in the previous arguments. The names of variable grades must match their definition. By this argument, the function counts the number of times a profile is observed in the population, assigning the result to the freq output. This method should be used when the variables and their grades are known, otherwise the function pop2prof is available.

## Value

The function returns a S3 class object wprof, "weighted profiles", comprising the data.frame profiles and the vector of frequencies freq.

#### See Also

#### pop2prof, getzeta

#### Examples

```
# 2 variables with 2 modalities, frequencies detected from population
pop <- matrix(sample(1:2, 100, replace=TRUE), 50, 2)
var2prof(varlen=c(2, 2))
# 2 variables:
# - mood: 2 modalities
# - weather: 3 modalities
# 2*3 profiles and frequencies sampled from a Binomial distribution n = 10, p = 0.5
var <- list(
    mood = ordered(c("bad", "good"), levels = c("bad", "good")),
    weather = ordered(c("rainy", "cloudy", "sunny"), levels = c("rainy", "cloudy", "sunny"))
)
var2prof(var, freq = rbinom(2*3, 10, 0.5), labtype = "progressive")</pre>
```

vertices

Coordinates of the vertices of the Hasse diagram, representing the input cover relation.

## Description

The function computes the coordinates of the vertices of the Hasse diagram.

## Usage

vertices(C, shape = c("square", "circle", "equispaced"), noise = FALSE)

## Arguments

С	cover matrix, an object of class S3 cover.
shape	shape of the diagram. See details.
noise	some jittering on the x axis, so as to improve readability. Values can be boolean
	or positive values, to get different jittering intensities.

## Details

Possible Hasse diagram shapes: square; circle; equispaced. The last option is suggested when the poset has more than one maximal or minimal elements. The function is used by the plot methods defined in the package (see plot.cover).

#### See Also

plot.cover

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